MATH 1ZC3/1B03: Test 1 - Version 1
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Duration: 90 min.

Name: SOLUTIONS ID #: ______________

Instructions:

This test paper contains 23 multiple choice questions printed on both sides of the page. The questions are on pages 2 through 24. Pages 25 to 26 are available for rough work. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF THE INVIGILATOR.

Select the one correct answer to each question and ENTER THAT ANSWER INTO THE SCAN CARD PROVIDED USING AN HB PENCIL. Room for rough work has been provided in this question booklet. You are required to submit this booklet along with your answer sheet. HOWEVER, NO MARKS WILL BE GIVEN FOR THE WORK IN THIS BOOKLET. Only the answers on the scan card count for credit. Each question is worth 1 mark. The test is graded out of 23. There is no penalty for incorrect answers. **NO CALCULATORS ALLOWED.**

Computer Card Instructions:

IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED. YOUR TEST RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner that will read the answer sheets senses areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 **and fill the corresponding bubbles underneath.**
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked "1".

1. Which of the following matrices are in reduced row echelon form?

(i) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ✓

(ii) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ ✗ There is a 1 below a leading 1

(iii) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ✓

(iv) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ✓

(v) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ ✗ There is a 1 below and to the left of a leading 1

(a) (i), (ii), (iii) only

(b) All of the them

(c) None of them

(d) (i), (iii), (iv) only

(e) (iii) only

2. Find the general solution of the following linear system (in three unknowns x_1, x_2, x_3):

$$\begin{array}{rcrcrcl} & & 4x_2 & + & 8x_3 & = & 0 \\ & & 2x_2 & + & 4x_3 & = & 0 \\ 5x_1 & + & 10x_2 & + & 15x_3 & = & 0 \end{array}$$

$a) \begin{cases} x_1 = t+1 \\ x_2 = -t \\ x_3 = t \end{cases}$ $b) \begin{cases} x_1 = 2t \\ x_2 = 3t \\ x_3 = 4t \end{cases}$ $\boxed{c) \begin{cases} x_1 = t \\ x_2 = -2t \\ x_3 = t \end{cases}}$

$d) \begin{cases} x_1 = t \\ x_2 = 2t \\ x_3 = 1 \end{cases}$ $e) \begin{cases} x_1 = 2t \\ x_2 = 0 \\ x_3 = t \end{cases}$

$$\left(\begin{array}{ccc|c} 0 & 4 & 8 & 0 \\ 0 & 2 & 4 & 0 \\ 5 & 10 & 15 & 0 \end{array}\right) \xrightarrow{\text{Row operations}} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

We get $x_1 - x_3 = 0$ and $x_2 + 2x_3 = 0$.

Setting $x_3 = t$, we obtain

$$x_1 = t$$
$$x_2 = -2t$$
$$x_3 = t$$

3. Without using row reduction, determine how many solutions the following homogeneous linear system has

$$\begin{array}{rcrcrcrcl} \#\#x_1 & + & 33x_2 & + & 8x_3 & + & x_4 & = & 0 \\ 20x_1 & + & 22x_2 & + & 9x_3 & + & 2x_4 & = & 0 \\ 300x_1 & + & 7x_2 & + & 11x_3 & + & 3x_4 & = & 0 \end{array}$$

where ## is a number that is not given to you.

(a) One

(b) None

(c) Two

(d) $\boxed{\text{Infinitely many}}$

(e) One and a half

Any homogeneous linear system with more unknowns than equations has infinitely solutions.

(This is because the number of leading variables is always bounded by the number of equations, and the rest of the variables will be free, yielding infinitely many solutions)

4. Let I be the 3×3 identity matrix and

$$J = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Which of the following are true?

(i) $J^3 = 2J$ (ii) $J^2 = 2\,\mathrm{I}$ (iii) J is invertible

(a) (i), (ii) only
$\boxed{\text{(b) (i) only}}$
(c) (ii) only
(d) None of them
(e) All of them

- Row 1 and 3 of J are equal, so $\det J = 0$. Thus, J is <u>not</u> invertible.
- $J^2 = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, so $J^2 \neq 2I$
- $J^3 = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 0 \end{pmatrix}$, so $J^3 = 2J$

5. Find all of the values of constants a and b for which the following linear system (in three unknowns x_1, x_2, x_3) is consistent.

$$\begin{array}{rcrcrcl} x_1 & + & ax_2 & + & a^2x_3 & = & 1 \\ ax_1 & + & a^2x_2 & + & a^3x_3 & = & 1 \\ a^2x_1 & + & a^3x_2 & + & a^4x_3 & = & a^2+b \end{array}$$

(a) $a = 1$ and $b = 0$

(b) $a = 0$ and $b = 1$

(c) $a = 0$ and $b = 0$

(d) $a =$ any real number and $b = -1$

(e) $a \geq 0$ and $b \geq 0$

$$\left(\begin{array}{ccc|c} 1 & a & a^2 & 1 \\ a & a^2 & a^3 & 1 \\ a^2 & a^3 & a^4 & a^2+b \end{array}\right) \overset{\text{Row operations}}{\sim} \left(\begin{array}{ccc|c} 1 & a & a^2 & 1 \\ 0 & 0 & 0 & 1-a \\ 0 & 0 & 0 & b \end{array}\right)$$

The system being consistent is equivalent to the last 2 rows being zero. Thus, we get $a=1$ and $b=0$.

6. Let A and B be square matrices of the same size. Let c and d be scalars. Recall that $\mathrm{tr}(A)$ denotes the trace of A, and A^T the transpose of A. Which of the following statements are always true?

   (i) $\mathrm{tr}(AB) = \mathrm{tr}(A) \cdot \mathrm{tr}(B)$

   (ii) $\mathrm{tr}(cA + dB) = c\,\mathrm{tr}(A) + d\,\mathrm{tr}(B)$

   (iii) $\mathrm{tr}\left(\frac{1}{2}A + \frac{1}{2}A^T\right) = \mathrm{tr}(A)$

   (a) None of them

   (b) (i), (ii) only

   (c) (ii), (iii) only

   (d) All of them only

   (e) (i) only

(i) is not always true: For example, if $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = B$, then $tr(AB) = 2$ but $tr(A) \cdot tr(B) = 2 \cdot 2 = 4$.

(ii) Recall that $tr(cA) = c\,tr(A)$ and $tr(A+B) = tr(A) + tr(B)$, this implies $tr(cA + dB) = tr(cA) + tr(dB) = c\,tr(A) + d\,tr(B)$. ✓

(iii) Recall that $tr(A^T) = tr(A)$, this implies

$$tr\left(\tfrac{1}{2}A + \tfrac{1}{2}A^T\right) = \tfrac{1}{2}tr(A) + \tfrac{1}{2}tr(A^T) = \tfrac{1}{2}tr(A) + \tfrac{1}{2}tr(A)$$
$$= tr(A) \checkmark$$

7. Which of the following matrices are NOT elementary matrices?

(i) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ ✓ Obtained from I_4 by interchanging rows 2 and 4.

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ X This not a square matrix

(iii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ ✓ Obtained from I_2 by interchanging row 1 and 2

(iv) $\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ ✓ Obtained from I_2 by adding row 1 multiplied by 3 to row 2.

(v) $\begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}$ X Need <u>at least 2</u> row operations applied to I_2.

(a) (i), (ii) only

(b) (ii), (iv) only

(c) None of them

✓ $\boxed{\text{(d) (ii), (v) only}}$

(e) (iii) only

8. Let

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}.$$

Which of the following is the first row of A^{-1}?

(a) $\begin{bmatrix} -3 & 1 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -3/2 & -1/2 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} -3/2 & 1/2 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & -2 \end{bmatrix}$

$$\left(\begin{array}{ccc|ccc} 0 & 2 & 1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 1 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 2 & 0 & 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & -3 & 1 & 6 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -3/2 & 1/2 & 3 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & -2 \end{array}\right)$$

9. Let A be a square matrix of order n. Which one of the following statements is <u>not</u> equivalent to the others?

   (a) $Ax = 0$ has no nontrivial solution.

   (b) $Ax = b$ has no solution or infinitely many solutions for any $n \times 1$ matrix b

   (c) A is singular.

   (d) A is not expressible as a product of elementary matrices.

   (e) The reduced row echelon form of A is not an identity matrix.

Recall from the theorem we have been building up in class that (b) to (e) are all equivalent, and they are also equivalent to $A\bar{x} = 0$ having at least one non-trivial solution.

10. Find the determinant of the matrix:

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 7 & -2 & 0 & 0 \\ 1 & 5 & 1 & 0 \\ 0 & 2 & 6 & 9 \end{bmatrix}$$

(a) 10

(b) -18

(c) 5

(d) 2

$\boxed{\text{(e) -36}}$

The determinant of a triangular matrix is the product of its diagonal entries.

11. Given

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a & b & a \\ c & d & c \\ 2 & -1 & 3 \end{bmatrix},$$

if $\det(A) = 4$, what is $\det(B)$?

$\boxed{\text{(a) } 4}$

(b) 20

(c) -12

(d) 0

(e) There is insufficient information

$$\det B = 2 \det \begin{pmatrix} b & a \\ d & c \end{pmatrix} + \underbrace{\det \begin{pmatrix} a & a \\ c & c \end{pmatrix}}_{0} + 3 \det \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= -2 \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} + 3 \det \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det A = 4.$$

12. Let M be the 3×3 matrix $M = \begin{bmatrix} 3 & 0 & 2 \\ 0 & -1 & 2 \\ 2 & 5 & 1 \end{bmatrix}$. Find the cofactor, C_{21} of this matrix.

(a) 4

(b) 0

(c) -4

(d) 10

(e) -10

$$C_{21} = (-1)^{2+1} \det \begin{pmatrix} 0 & 2 \\ 5 & 1 \end{pmatrix}$$

$$= (-1) \cdot (-10) = 10$$

13. Given

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4g & 4h & 4i \\ a+3g & b+3h & c+3i \\ -d & -e & -f \end{bmatrix},$$

if $\det(A) = 12$, what is $\det(B)$?

(a) 3

(b) -1

(c) 36

(d) 12

$\boxed{\text{(e) -48}}$

$$\det B = \det\begin{pmatrix} 4g & 4h & 4i \\ a+3g & b+3h & c+3i \\ -d & -e & -f \end{pmatrix}$$

$$\begin{pmatrix} \text{Interchange} \\ R_1 \longleftrightarrow R_3 \\ \text{and then} \\ R_1 \longleftrightarrow R_2 \end{pmatrix} = \det\begin{pmatrix} a+3g & b+3h & c+3i \\ -d & -e & -f \\ 4g & 4h & 4i \end{pmatrix}$$

$$\begin{pmatrix} \text{factor out} \\ 4 \text{ from } R_3 \\ \text{and } -1 \text{ from} \\ R_2 \end{pmatrix} = -4 \det\begin{pmatrix} a+3g & b+3h & c+3i \\ d & e & f \\ g & h & i \end{pmatrix}$$

$$\boxed{-48}$$

$$\begin{pmatrix} \text{add } (-3)\cdot R_3 \\ \text{to } R_1 \end{pmatrix} = -4 \det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = (-4)\cdot \det A = (-4)\cdot(12)$$

14. Consider the 3×3 matrix $A = \begin{bmatrix} a & 2b-1 & a^3 \\ b^2 & 4 & 3 \\ 8 & 3 & 4a \end{bmatrix}$. Find the values of a and b such that A is symmetric.

(a) $a = 8, b = 2$

(b) $a = 5, b = 3$

(c) $a = b = 0$

$\boxed{\text{(d) } a = 2, b = 1}$

(e) No such values exist.

For A to be symmetric we must have

$b^2 = 2b - 1$ and $8 = a^3$.

We get $b^2 - 2b + 1 = 0$

$(b-1)^2 = 0$

so $b = 1$.

Also, $a^3 = 8$ implies $a = 2$.

15. Given the 3x3 matries A, B, C such that $\det A$=2, $\det B$=5, and $\det C$=−3, evaluate $\det(2CAB^TC^{-1})$

(a) 16/5

(b) 36/5

(c) -20

(d) 88/3

$\boxed{\text{(e) 80}}$

$$\det(2CAB^TC^{-1}) = 2^3 \cdot \det C \cdot \det A \cdot \det(B^T) \cdot \det(C^{-1})$$

$$= 2^3 \cdot \cancel{\det C} \cdot \det A \cdot \det B \cdot \left(\frac{1}{\cancel{\det C}}\right)$$

$$= 2^3 \cdot \det A \cdot \det B$$

$$= 2^3 \cdot 2 \cdot 5 = 80$$

16. Find the eigenvalues of

$$A = \begin{bmatrix} -1 & 6 \\ -3 & 8 \end{bmatrix}$$

(a) 3 and 5

(b) -2 and 6

$\boxed{\text{(c) 2 and 5}}$

(d) -5 and 1

(e) 3 and 4

$$p_A(t) = \det(tI - A) = \det\begin{pmatrix} t+1 & -6 \\ 3 & t-8 \end{pmatrix}$$

$$= (t+1)(t-8) + 18$$

$$= t^2 - 7t + 10$$

$$= (t-5)(t-2).$$

So $\lambda_1 = 5$ and $\lambda_2 = 2$.

17. Find an eigenvector associated with the eigenvalue of 2 from the matrix

$$\begin{bmatrix} 4 & -1 & -1 \\ 5 & -2 & 2 \\ -3 & 3 & -1 \end{bmatrix}$$

(a) $[\,1,\ 1,\ 0\,]^T$
(b) $[\,-1,\ -2,\ 3\,]^T$
(c) $[\,2,\ 3,\ 1\,]^T$
(d) $[\,0,\ -1,\ 1\,]^T$
(e) $[\,2,\ 1,\ 2\,]^T$

We have

- $$\begin{pmatrix} 4 & -1 & -1 \\ 5 & -2 & 2 \\ -3 & 3 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \neq 2\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

- $$\begin{pmatrix} 4 & -1 & -1 \\ 5 & -2 & 2 \\ -3 & 3 & -1 \end{pmatrix}\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ * \\ * \end{pmatrix} \neq 2\begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix}$$

- $$\begin{pmatrix} 4 & -1 & -1 \\ 5 & -2 & 2 \\ -3 & 3 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix} = 2\underbrace{\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}}_{\text{eigenvector of } 2.} \quad \checkmark$$

18. For what two values of k is

$$\begin{bmatrix} 5 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 1 & k \end{bmatrix}$$

not diagonalizable?

(a) $k = 0$ and $k = 1$

(b) $k = 3$ and $k = 5$

(c) $k = 0$ and $k = 7$

(d) $k = 2$ and $k = 5$

(e) $k = -7$ and $k = 1$

For any choice other than (d), the matrix will have 3 different eigenvalues, and so it would be diagonalizable.

19. The matrix A has eigenvalues -1 and 1, with corresponding eigenvectors of $[2,1]^T$ and $[0,1]$ respectively. What is A^{100}?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & -1 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 0 \\ -1 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix}$

$$A^{100} = P D^{100} P^{-1}$$

where $P = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$ and $D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$,

so

$$A^{100} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} (-1)^{100} & 0 \\ 0 & 1^{100} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

20. You are given that the matrix

$$P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

diagonalizes the 2×2 matrix A. Which of the following also diagonalizes A?

(a) $\begin{bmatrix} 10 & 2 \\ 30 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 20 \\ 6 & 40 \end{bmatrix}$

(e) all of the above

Since $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ are eigenvectors of A, any multiple of each is also an eigenvector, and moreover they will also diagonalize A. Finally, in all of the above matrices, the fist column is proportional to $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and the second column is proportional to $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

21. You are given that the matrix A has eigenvectors of $\mathbf{x_1} = [1,0,1]^T$ and $\mathbf{x_2} = [0,1,1]^T$, so that

$$A\mathbf{x_1} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} \quad \text{and} \quad A\mathbf{x_2} = \begin{bmatrix} 0 \\ 10 \\ 10 \end{bmatrix}$$

Notice that $\mathbf{x_1} + \mathbf{x_2} = [1,1,2]^T$. If

$$A^4 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

what is c?

(a) 20,000

(b) 12^4

(c) 16,000

(d) 10,000

$\boxed{\text{(e) 10,016}}$

From $A\bar{x}_1 = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, we see that $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ is an eigenvector of the eigenvalue 2.

From $A\bar{x}_2 = \begin{pmatrix} 0 \\ 10 \\ 10 \end{pmatrix} = 10\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, we see that $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of the eigenvalue 10.

We get

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = A^4\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = A^4\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + A^4\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 2^4\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + 10^4\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2^4 \\ 10^4 \\ 2^4+10^4 \end{pmatrix}, \quad \text{and so } c = 2^4 + 10^4 = 10{,}016.$$

22. Let A, B, C be square matrices of order n, I be the identity matrix of order n and k be a real number. Which one of the following statements is true?

(a) $AB = 0$ implies $A = 0$ or $B = 0$.

(b) $A^2 = A$ implies $A = 0$ or $A = I$.

(c) $AB = AC$ and $A \neq 0$ implies $B = C$.

(d) $(kA)^T = 0$ implies $k = 0$ or $A = 0$.

(e) There are exactly 2 solutions of this matrix equation $A^2 = I$.

(a) Not always true: Choose $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = B$, then $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

(b) Not always true: Choose $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, then $A^2 = A$.

(c) Not always true: Choose $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = B$ and $C = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

then $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = AC$.

d) Always true: If $(kA)^T = 0$, then $k \cdot A^T = 0$. The latter implies that $k = 0$ or $A^T = 0$. But since $A^T = 0 \Longleftrightarrow A = 0$, we get that either $k = 0$ or $A = 0$.

e) Not true: Choosing $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, or $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$, or $A = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

all yield $A^2 = I$.

23. Suppose that A is a 15×15 matrix, and we want to extract a submatrix consisting of rows 8 to 12, and columns 11 to 15 of A. What single Matlab command (using 10 characters or less) could accomplish this?

   (a) A(1:5,2:3)
   (b) A(8:12,11:15)
   (c) SUBMAT(8:12,11:15)
   (d) ARowCol(8:12,11:15)
   (e) RREF(1,15)

END OF TEST QUESTIONS