Math 1B03 Final Exam Review

December 12, 2016

This is a selection of questions on topics that could be on the final exam. The topics covered here are not exhaustive, but do highlight some of the important parts of the course.

I'll be in the math help centre tomorrow (Dec. 13) from 2:30pm - 4:30pm for any last questions. It may be busy, so please come prepared with specific questions if you can (e.g. an exercise from the book or something like, "How do I tell if a vector is in the span of these other vectors?"). If many people are there, I might not have time to explain a topic from start to finish (e.g. a question like, "I don't get vector spaces").

1. Consider the matrix,
$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -3 \\ 0 & -2 & -3 \end{bmatrix}$$
Find the second row of A^{-1}.

2. Write down the elementary matrices needed to produce,
$$\begin{bmatrix} 1 & 0 & 4 \\ -2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 0 & -1 & 3 \\ -1 & -1 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

3. Let A be the matrix,
$$\begin{bmatrix} 7 & 0 & -3 \\ -9 & -2 & 3 \\ 18 & 0 & -8 \end{bmatrix}$$
   (a) What are the eigenvalues of A?
   (b) Find a basis for the eigenspace corresponding to the least eigenvalue.
   (c) What are the eigenvalues of A^{-1}?

4. For the matrix,
$$A = \begin{bmatrix} 4 & 2 & 3 & 3 \\ 0 & 2 & k & 3 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$
find k so that the the algebraic and geometric multiplicity of $\lambda = 4$ is the same.

5. (a) Diagonalize the matrix,
$$A = \begin{bmatrix} 3 & 0 \\ 4 & 7 \end{bmatrix}$$

(b) If B is matrix that is similar to A, is B also diagonalizable?

6. If $\det\left(3(A^T)^{-1}BCB^{-1}\right) = \det\left(2I^{-1}\right)$, $\det(B) = 3$, $\det(C) = 4$, and all matrices are 2×2, find $\det(A)$.

7. Every Tuesday after tutorial, Sean always stops at either a grocery store, coffee shop, or bakery on the way home. If he goes to a grocery store one Tuesday, then he doesn't go to one the next week, but there's an equal chance he goes to a coffee shop or a bakery. If he goes to a coffee shop or bakery, the next week he has an equal chance of going to any of the three.

   (a) If he went to a grocery store last week and today is Tuesday, what is the probability that he'll go to a bakery next Tuesday?

   (b) Which of the three will he tend to go to most over the long-run?

8. If A is invertible, which of the following are always true:

   i. If A is diagonalizable, then A^{-1} is diagonalizable

   ii. A is diagonalizable

   iii. 0 is not an eigenvalue of A

   iv. $\det(A) > 0$

9. Find the area of the triangle with vertices at (-5,-5), (-4,-1), and (-1,-4).

10. Let $\vec{w} = (4, 4)$ and $\vec{v} = (1 - \sqrt{3}, 1 + \sqrt{3})$.

    (a) Find the component of $\vec{v}$ orthogonal to $\vec{w}$.

    (b) What is the angle between $\vec{v}$ and $\vec{w}$?

11. Suppose that $\vec{u}, \vec{v} \in \mathbb{R}^3$. Which of the following are always orthogonal to $\vec{u} + \vec{v}$?

    i. $\vec{u} \times \vec{v}$

    ii. $(\vec{u} \circ \vec{v})\vec{v}$

    iii. $\vec{u} \times \vec{w} + \vec{v} \times \vec{w}$, where $\vec{w}$ is any vector in $\mathbb{R}^3$

12. Find z if,

$$\frac{2-i}{-1-i} + \frac{z}{1+2i} = 4$$

13. Write $z = \sqrt{3} + 3i$ in polar form.

14. The set,

$$\{(1,0,1), (5,-1,1), (5,2,-2)\}$$

    is a basis for $\mathbb{R}^3$. The first two vectors found using the Gram-Schmidt process are $(1, 0, 1)$ and $(2, -1, -2)$. Complete the process and find an orthonormal basis.

15. Take $\vec{v}_1 = (1, 1, 1)$, $\vec{v}_2 = (1, 2, 3)$, and $\vec{v}_3 = (1, 1, 2)$.

    (a) Is this a basis for $\mathbb{R}^3$?

    (b) If so, write $(4, -1, 0)$ in coordinates relative to the basis.

16. Is the set,

$$\{2 - x + x^2, 3 - 4x - 2x^2, 5 - 10x - 8x^2\}$$

    linearly independent?

17. Consider the matrix,

$$\begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix}$$

   (a) Find a basis for the null space of A

   (b) Find a basis for the row space of A

   (c) Find a basis for the column space of A

18. (a) Recall that $F(-\infty, \infty)$ is the vector space consisting of all functions defined on $\mathbb{R}$ with the usual addition and scalar multiplication. Which of the following are subspaces of $F(-\infty, \infty)$?

      i. $W = \{\text{functions s.t.} f(-x) = f(x)\}$

      ii. $V = \{\text{functions s.t.} f(-x) = 1 + f(x)\}$

   (b) Is the set of all 2×2 elementary matrices a subspace of M_{22}?

19. Which of the following are in the span of,

$$\{(1, -3, -2), (-2, 7, 4), (3, -8, -6)\}$$

   i. $[1, 1, -2]^T$

   ii. $[-2, 1, 1]^T$

   iii. $[-3/2, 1/2, 3]^T$

20. For an $n \times n$ matrix A, which of the following are always true,

   i. If $\dim(\text{null}(A)) = \dim(\text{row}(A))$, then n is even

   ii. If n is odd and A is skew-symmmetric, then A is not invertible

   iii. If $\{\vec{v}_1, \cdots, \vec{v}_k\}$ spans $\text{col}(A)$, then it also spans $\text{row}(A)$

   iv. If $\{\vec{v}_1, \vec{v}_2\}$ is a basis for $\text{null}(A)$ and $\vec{w} \in \mathbb{R}^n$ is a solution to $A\vec{x} = \vec{b}$, then every solution can be written as $\vec{w} + t\vec{v}_1 + s\vec{v}_2$

21. Select a correct answer,

   (a) If A is a 3×5 matrix and $\text{rank}(A) = 3$, then $Ax = b$ (always/sometimes, but not always/never) has (a unique solution/infinitely many solutions/no solutions)

   (b) If A is a 5×3 matrix and $\text{rank}(A) = 3$, then $Ax = b$ (always/sometimes, but not always/never) has (a unique solution/infinitely many solutions/no solutions)