MATH 1B03: Midterm 1 - VERSION 1
Instructor: Adam Van Tuyl
Date: October 6, 2016 7:00PM
Duration: 75 min.

Name: SOLUTIONS ID #: ______________

Instructions:
This test paper contains 21 multiple choice questions printed on both sides of the page. The questions are on pages 2 through 9. Scrap paper is available for rough work. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCIES TO THE ATTENTION OF THE INVIGILATOR.

Select the one correct answer to each question and ENTER THAT ANSWER INTO THE SCAN CARD PROVIDED USING AN HB PENCIL. You are required to submit this booklet along with your answer sheet. HOWEVER, NO MARKS WILL BE GIVEN FOR THE WORK IN THIS BOOKLET. Only the answers on the scan card count for credit. Each question is worth 1 mark. The test is graded out of 21. There is no penalty for incorrect answers.

NO CALCULATORS ALLOWED.

Computer Card Instructions:

IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED. YOUR TEST RESULTS DEPEND UPON PROPER ATTENTION TO THESE INSTRUCTIONS.

The scanner that will read the answer sheets senses areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will **NOT** be sensed. Erasures must be thorough or the scanner may still sense a mark. Do **NOT** use correction fluid.

- Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet **MUST** be signed in the space marked SIGNATURE.
- Mark your student number in the space provided on the sheet on Side 1 **and fill the corresponding bubbles underneath**. Your student number **MUST** be 9 digits long. If you have a student number that is 7 digits, begin your student number with 00 (two zeroes).
- Mark only **ONE** choice (A, B, C, D, E) for each question.
- Begin answering questions using the first set of bubbles, marked "1".

1. Which equation is **NOT** linear in x_1, x_2 and x_3.

(a) $x_1 + x_2 + x_3 = 2016.$

(b) $(\sin(2016))x_1 + (\cos(2016))x_3 = 0.$

(c) $\sqrt{2016}x_1 + \pi^{2016}x_2 + e^{2016}x_3 = 42.$

(d) $x_1^{2016} + \sin x_2 + \log_{10}(x_3) = 2016.$ not linear

(e) $-x_1 - 2x_2 - 3x_3 - 4 = 0.$

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2. What is the augmented matrix for the following system of 3 linear equations in 6 unknowns?

$$\begin{aligned} x_1 + 4x_3 - \pi x_5 &= 42 \\ (\cos(3))x_2 - 6x_4 - 11x_6 &= 0 \\ 3x_2 + 3x_3 + 17x_5 &= \log_{10} 8 \end{aligned}$$

(a) $\left[\begin{array}{ccc|c} 1 & 4 & -\pi & 42 \\ \cos(3) & -6 & -11 & 0 \\ 3 & 3 & 17 & \log_{10} 8 \end{array}\right]$ X not a 3×6 matrix

(b) $\left[\begin{array}{ccccc|c} 1 & 0 & 4 & 0 & -\pi & 42 \\ 0 & \cos(3) & -6 & -11 & 0 & 0 \\ 0 & 3 & 3 & 0 & 17 & \log_{10} 8 \end{array}\right]$ X not a 3x6 matrix

(c) $\left[\begin{array}{cccccc|c} -1 & 0 & -4 & 0 & \pi & 0 & 42 \\ 0 & \cos(3) & 0 & -6 & 0 & -11 & 0 \\ 0 & 3 & 3 & 0 & 17 & 0 & \log_{10} 8 \end{array}\right]$ X coeffecient of x_1 wrong in first eq

(d) $\left[\begin{array}{cccccc|c} 1 & 0 & 4 & 0 & -\pi & 0 & 42 \\ 0 & \cos(3) & 0 & -6 & 0 & -11 & 0 \\ 0 & 3 & 3 & 0 & 17 & 0 & \log_{10} 8 \end{array}\right]$

(e) This is not a system of linear equations X this is a SLE

3. Which of the following matrices are in reduced row echelon form?

not reduced — not reduced — not reduced — not echelon

i) $\begin{bmatrix} 1 & \cancel{3} & 0 & 0 & 3 \\ 0 & 1 & \cancel{0} & 2 & -2 \\ 0 & 0 & 1 & -9 & 8 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$ ii) $\begin{bmatrix} 0 & 1 & -5 & 0 & 0 \\ 0 & 0 & 0 & \pi & 6 \\ 0 & 0 & \cancel{0} & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ iii) $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \cancel{0} & 1 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$ iv) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & \cancel{42} & 0 \\ 0 & 0 & 0 \end{bmatrix}$

v) $\begin{bmatrix} 0 & 1 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

(a) iii) and v) only
(b) All of them
(c) None of them
(d) v) and i) only
(e) v) only

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4. The following system of linear equations has how many solutions?

$$-2x_1 + 3x_2 - 3x_3 = -9$$
$$3x_1 - 4x_2 + x_3 = 5$$
$$-5x_1 + 7x_2 + 2x_3 = -14$$

(a) None
(b) $\boxed{\text{One}}$
(c) Two ✗ not possible
(d) 2016 ✗ not possible
(e) Infinitely many

$$\begin{bmatrix} -2 & 3 & -3 & -9 \\ 3 & -4 & 1 & 5 \\ -5 & 7 & 2 & -14 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 3 & 9 \\ 3 & -4 & 1 & 5 \\ -5 & 7 & 2 & -14 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 3 & 9 \\ 3 & -4 & 1 & 5 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 6 & -9 & 9 & 27 \\ 6 & -8 & 2 & 10 \\ 0 & 0 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 6 & -9 & 9 & 27 \\ 0 & 1 & -7 & -17 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$

← from this matrix, can see we only have 1 sol^n

5. The **rank** of a matrix A is the number of leading 1's in the reduced row echelon form of A. What is the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 1 & 2-a & -1 & -2a^2 \end{bmatrix}$$

(a) 0

(b) 1

(c) 2

(d) 3

(e) Not enough information; answer will depend upon the value of a.

$$A \sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 0 & -a & 0 & -2a^2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 0 & 0 & 1-a & a^2-2a+1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & a & 1-a & a^2+1 \\ 0 & 0 & 1-a & -a^2+1 \end{bmatrix}$$

Note that if $a=1$, bottom row is zero, so rank is 2.
If $a \neq 0$ or $a \neq 1$, rank is 3.
So (e) is answer

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6. You are given the following three matrices

$$A = \begin{bmatrix} 5 & -6 & 4 & -4 \\ -8 & 9 & -2 & 3 \\ -4 & 7 & 3 & -1 \end{bmatrix}, B = \begin{bmatrix} -8 & 9 & -2 & 3 \\ 8 & -5 & 4 & -1 \\ -2 & 5 & -3 & 2 \end{bmatrix}, C = \begin{bmatrix} -4 & 9 \\ 6 & -5 \\ -9 & 3 \\ -1 & 5 \end{bmatrix}$$

What matrix multiplication will yield a 2×2 matrix?

(a) $CABC$

(b) $\boxed{C^TA^TBC}$

(c) $A^TB^TAC^T$

(d) BC^TA^TB

(e) ABA^TC

A is 3×4, B is 3×4 and C is 4×2
A^T is 4×3, B^T is 4×3 and C^T is 2×4

So, to get a 2×2 matrix, the first matrix we ~~must~~ need to begin ~~multiply~~ our matrix mult. with C^T and end with C.
This means (b) is only option.

7. Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and when multiplied together, they commute. Which of the following must necessarily be true?

(a) $a = 0$

$\boxed{\text{(b) } b = 0}$

(c) c can be any number

(d) $d = 0$

(e) none of the above

Commute means $AB = BA$.

$AB = \begin{bmatrix} a & 0 \\ c & 0 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} = BA$

So $b=0$ must be true and $c=0$ must be true. Only "$b=0$ must be true" is our list of options

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8. If A, B, C and D are invertible matrices of the same size and

$$(AB)^{-1}C^TA^{-1} = D$$

which of the following must be B?

(a) $AC^TA^{-1}D^{-1}$

$\boxed{\text{(b) } A^{-1}C^TA^{-1}D^{-1}}$

(c) $A^{-1}C^TAD^{-1}$

(d) $A^{-1}CA^{-1}D^{-1}$

(e) $A^{-1}C^TA^{-1}D^T$

$(AB)^{-1}C^TA^{-1} = D$

$B^{-1}A^{-1}C^TA^{-1} = D$

$B(B^{-1}A^{-1}C^TA^{-1})D^{-1} = BDD^{-1}$

$\boxed{A^{-1}C^TA^{-1}D^{-1} = B}$

9. Compute A if $(B+3A)^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$.

(a) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$

(e) $\boxed{\begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}}$

$(B+3A)^{-1} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} \Rightarrow B+3A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}^{-1}$

$= \frac{1}{8-9}\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

So $B+3A = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

Thus $3A = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} - B = \begin{bmatrix} -6 & 3 \\ 0 & -3 \end{bmatrix}$

So $A = \frac{1}{3}\begin{bmatrix} -6 & 3 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix}$

10. Which one of the following statements is not equivalent to the others?

(a) A is invertible.

(b) $A\mathbf{x} = \mathbf{0}$ has only one solution.

(c) The reduced row echelon form of A has a row of zeros.

(d) A is a product of elementary matrices.

(e) $A\mathbf{x} = \mathbf{b}$ is consistent for every $n \times 1$ matrix $\mathbf{b}$.

Should read: "reduced row echelon form of A ~~b~~ is I_n"

11. Let k be a nonzero number. What is the inverse of the matrix

$$\begin{bmatrix} k & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & k \end{bmatrix}$$

(a) $\begin{bmatrix} \frac{1}{k} & \frac{1}{k} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{k} \end{bmatrix}$

(b) $\boxed{\begin{bmatrix} \frac{1}{k} & -\frac{1}{k} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{k} \end{bmatrix}}$

(c) $\begin{bmatrix} \frac{1}{k} & -\frac{1}{k} & 0 \\ 0 & k & 0 \\ 0 & 0 & \frac{1}{k} \end{bmatrix}$

(d) $\begin{bmatrix} k & -\frac{1}{k} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{k} \end{bmatrix}$

(e) $\begin{bmatrix} -\frac{1}{k} & -\frac{1}{k} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{k} \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} k & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & k & 0 & 0 & 1 \end{array}\right] \sim \begin{bmatrix} k & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & k & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/k & -1/k & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/k \end{array}\right]$$

Inverse

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12. $A = \begin{bmatrix} 2 & 0 & 1 \\ 6 & 4 & 2 \end{bmatrix}$ is transformed into $B = \begin{bmatrix} 6 & 4 & 2 \\ 14 & 8 & 5 \end{bmatrix}$ by two elementary row operations, the first operation of which involves swapping rows. What are the corresponding elementary matrices E_1 and E_2?

(a) $\boxed{E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } E_2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}}$

(b) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $E_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

(d) $E_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(e) $E_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $E_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

E_1 must be swapping rows

This is the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

So only (a) has this option

13. The system of 5 equations in 4 unknowns $A\mathbf{x} = B$ has solutions

$$\mathbf{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

If performing row operations on the augmented matrix $[A|B]$ can produce the following matrix

$$\left[\begin{array}{cccc|c} 6 & 18 & 1 & -11 & 7 \\ -4 & -12 & 6 & 14 & a \\ -2 & -6 & -1 & 3 & b \\ -1 & -3 & 5 & 7 & c \\ 9 & 27 & -1 & -19 & d \end{array}\right]$$

what is $a + d$?

Since $\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ is a sol^n, we have

(a) -5

(b) 15

(c) 27

(d) 10

(e) 2016

$$\begin{bmatrix} 6 & 18 & 1 & -11 \\ -4 & -12 & 6 & 14 \\ -2 & -6 & -1 & 3 \\ -1 & -3 & 5 & 7 \\ 9 & 27 & -1 & -19 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ a \\ b \\ c \\ d \end{bmatrix}$$

This means $-12 + 14 = a$ and $27 - 19 = d$.

So $(a+d) = (-12+14) + (27-19)$
$= 2 + 8 = \boxed{10}$

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14. Find all the values of the unknown constants that make the matrix A symmetric.

$$A = \begin{bmatrix} 2 & a - 2b + c & 2a + b + c \\ 3 & 5 & -2 \\ 0 & a + c & 7 \end{bmatrix}$$

(a) $a = -\frac{9}{2}$, $b = -\frac{5}{2}$, $c = -\frac{13}{2}$.

(b) $a = \frac{9}{2}$, $b = \frac{5}{2}$, $c = -\frac{13}{2}$.

(c) $a = 9, b = -5, c = -13$.

(d) $a = \frac{9}{2}$, $b = -\frac{5}{2}$, $c = -\frac{13}{2}$.

(e) There is no choice of a, b and c that makes the matrix symmetric.

Solve the SLE

$$\left.\begin{aligned} a - 2b + c &= 3 \\ 2a + b + c &= 0 \\ a + c &= -2 \end{aligned}\right\}$$ using standard techniques, sol^n is (d)

The following questions are all **TRUE-FALSE** questions.

15. The linear system

$$\begin{aligned} x + y &= 5 \\ 3x + 3y &= k \end{aligned}$$

$\begin{bmatrix}1&1&5\\3&3&k\end{bmatrix} \sim \begin{bmatrix}1&1&5\\0&0&k-15\end{bmatrix}$

Sol^n will either have an infinite # sol^ns (if $k=15$) or no sol^ns

cannot have a unique solution, regardless of the value of k.

(a) TRUE (b) FALSE

16. If a matrix is in reduced row echelon form, then it is also in row echelon form.

(a) TRUE (b) FALSE

17. If A and B are $n \times n$ matrices, then $\operatorname{tr}(A+B) = \operatorname{tr}(A) + \operatorname{tr}(B)$.

(a) TRUE (b) FALSE

18. The sum of two invertible matrices of the same size must be invertible.

(a) TRUE (b) FALSE

$\begin{bmatrix}1&0\\0&1\end{bmatrix}$ and $\begin{bmatrix}-1&0\\0&-1\end{bmatrix}$ invertible, but $\begin{bmatrix}1&0\\0&1\end{bmatrix} + \begin{bmatrix}-1&0\\0&-1\end{bmatrix} = \begin{bmatrix}0&0\\0&0\end{bmatrix}$

not invertible

19. If A is a $n \times n$ invertible matrix, and E is an elementary $n \times n$ matrix, then EA^T is invertible.

A invertible $\Rightarrow$ A^T invertible.

(a) TRUE (b) FALSE

E invertible.

Product of invertible matrices invertible

20. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and if the linear system $A\mathbf{x} = \mathbf{b}$ has a unique solution for every $n \times 1$ matrix $\mathbf{b}$, then $ad - bc = 0$.

(a) TRUE (b) FALSE

$ad - bc \neq 0$

21. If A and B are $n \times n$ matrix such that $A + B$ is upper triangular, then A and B are upper triangular.

(a) TRUE (b) FALSE

$\begin{bmatrix}1&1\\1&1\end{bmatrix} + \begin{bmatrix}1&1\\-1&1\end{bmatrix} = \begin{bmatrix}2&2\\0&2\end{bmatrix}$

not upper triangular. upper triangular

END OF TEST PAPER