Categories, Higher Categories, and Spaces

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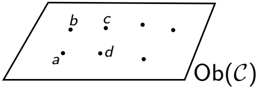
Definition of category

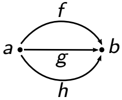
Category $\mathcal{C}$

Visualization

⊛ $a, b, c, d, \ldots \in \mathrm{Ob}(\mathcal{C})$

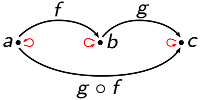


⊛ $f, g, h \in \mathrm{Hom}_{\mathcal{C}}(a, b)$



⊛ $\mathrm{id}_a \in \mathrm{Hom}_{\mathcal{C}}(a, a)$

⊛ $\mathrm{Hom}(b, c) \times \mathrm{Hom}(a, b) \xrightarrow{\circ} \mathrm{Hom}(a, c)$



Conditions

$$f \circ \mathrm{id}_a = f = \mathrm{id}_b \circ f \qquad (h \circ g) \circ f = h \circ (g \circ f)$$

Examples

- Top: Category of topological spaces and continuous maps
- Grp: Category of groups and group homomorphisms
- $\text{Vect}_{\mathbb{R}}$: Category of real vector spaces and linear transformations

Category $\mathcal{C}$ is called a *groupoid* if all of its morphisms are **isomorphisms**, i.e. for any $f : a \to b$, there exists $f^{-1} : b \to a$ with

$$f \circ f^{-1} = \text{id}_b \qquad f^{-1} \circ f = \text{id}_a$$

- Considering only homeomorphisms in Top $\rightsquigarrow$ groupoid $\underline{\text{Top}}$
- Considering only group isomorphisms in Grp $\rightsquigarrow$ groupoid $\underline{\text{Grp}}$

  Note that when $\mathcal{C}$ is a groupoid each $\text{Hom}(a, a)$ is a group.
- For any (discrete) group G there is a groupoid $\mathcal{G}$ with one object $\bullet$ and $\text{Hom}(\bullet, \bullet) \cong G$.

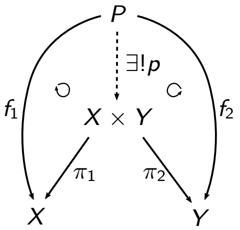
Why categories?

- Unifying common concepts in mathematics (limits, colimits,...)
- A different approach to foundations of mathematics (ETCS)
- Applications in other areas (computer science, physics, biology,...)
- Generalizations of concepts (categorification, sheaves,...)
- ...

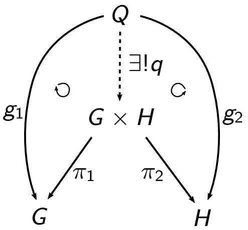
Abstractly unifying modern abstract mathematics

> *Category theory takes a bird's eye view of mathematics. From high in the sky, details become invisible, but we can spot patterns that were impossible to detect from ground level.*
>
> *Tom Leinster*

Consider a product $X \times Y$ of top. spaces and a product $G \times H$ of groups



$p(a) = (f_1(a), f_2(a))$



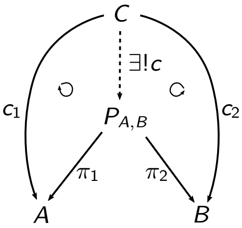
$q(a) = (g_1(a), g_2(a))$

Abstractly unifying modern abstract mathematics

Category theory takes a bird's eye view of mathematics. From high in the sky, details become invisible, but we can spot patterns that were impossible to detect from ground level.

Tom Leinster

A product of objects A and B in a category $\mathcal{C}$ is a triple $[P_{A,B}, \pi_1, \pi_2]$ where $\pi_1 : P_{A,B} \to A$ and $\pi_2 : P_{A,B} \to B$ such that for any triple $[C, c_1, c_2]$ there exists a unique map $c : C \to P_{A,B}$ commuting the diagram



⊛ Product of objects may not exists in a category, but when it is exists it is unique up to a unique isomorphism!

Functor category Func($\mathcal{C}, \mathcal{D}$)

How to relate/compare two categories? $\rightsquigarrow$ Functors

$$\begin{aligned} \mathcal{F} : \mathcal{C} &\to \mathcal{D} \\ a &\mapsto \mathcal{F}(a) \\ \big(f : a \to b\big) &\mapsto \big(\mathcal{F}(f) : \mathcal{F}(a) \to \mathcal{F}(b)\big) \end{aligned}$$

How to relate/compare functors? $\rightsquigarrow$ Natural transformations
$\eta : \mathcal{F} \to \mathcal{G}$ consists of $\{\eta_a : \mathcal{F}(a) \to \mathcal{G}(a)\}_{a \in \mathcal{C}}$ such that for any $f : a \to b$

$$\begin{array}{ccc} \mathcal{F}(a) & \xrightarrow{\mathcal{F}(f)} & \mathcal{F}(b) \\ {\scriptstyle \eta_a}\downarrow & \square & \downarrow{\scriptstyle \eta_b} \\ \mathcal{G}(a) & \xrightarrow[\mathcal{G}(f)]{} & \mathcal{G}(b) \end{array}$$

commutes. Natural transformations can be composed. Hence, for any categories $\mathcal{C}$ and $\mathcal{D}$ we have a new category $\mathrm{Func}(\mathcal{C}, \mathcal{D})$ of functors and natural transformations.

Categories inside a category??

Category

- Categories
- Functors between $\mathcal{C}$ and $\mathcal{D}$
- Natural transformations

Functors between $\mathcal{C}$ and $\mathcal{D}$ and Natural transformations: Category

Each $\mathrm{Hom}(\mathcal{C}, \mathcal{D})$ is a category.

$\Downarrow$

Objects

Morphisms

2-Morphisms

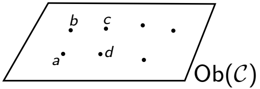
Morphisms between morphisms $\longrightarrow$ 2-Morphisms

Definition of 2-category

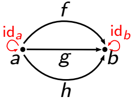
2-category $\mathcal{C}$

Visualization

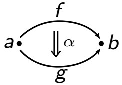
⊛ $a, b, c, d, \ldots \in \mathrm{Ob}(\mathcal{C})$



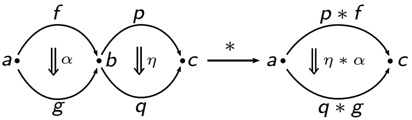
⊛ $\mathrm{Hom}_{\mathcal{C}}(a, b)$ is a category.

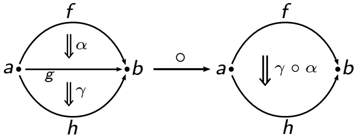


⊛ $\alpha, \beta, \gamma \in \mathrm{Hom}_{\mathcal{C}}(f, g)$



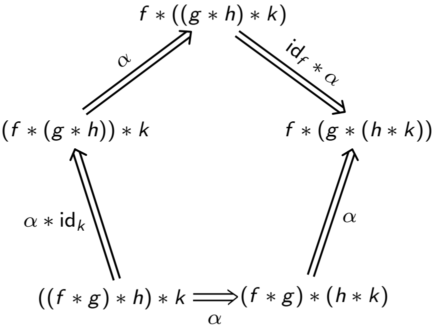
Compositions

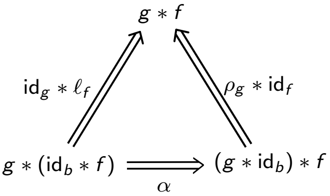




Definition of bicategory (Bénabou)

For every composable 1-morphisms f, g, h there exist isomorphisms $\alpha_{f,g,h} : (f * g) * h \to f * (g * h)$ such that the pentagon diagram commutes.





For any 1-morphism $f : a \to b$ there exist isomorphisms $\ell_f : id_b * f \to f$ and $\rho_f : f * \mathrm{id}_a \to f$ such that the triangle diagram commutes.

Examples of bicategories

Definition (Ehresmann)

A 2-*category* or a *strict bicategory* is a bicategory $(\mathcal{C}, \alpha, \ell, \rho)$ where the natural isomorphisms α, ℓ, and ρ are the identity isomorphisms.

Examples of bicategories

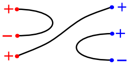
- Cat : categories, functors, and natural transformations
- $\mathsf{Alg}_{\Bbbk}$: $\Bbbk$-algebras, bimodules, and bimodule maps
  Composition of bimodules is given by

$$_C N_B \circ {_B M_A} = {_C N} \otimes_B M_A$$

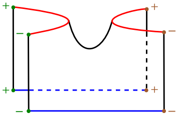
- Extended 2d-bordism bicategory Bord_2 (Schommer-Pries)

Objects

1-morphisms

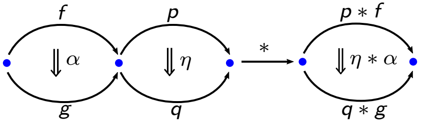


2-morphisms



What is a bicategory with one object?

A bicategory $(\mathcal{C}, \alpha, \ell, \rho)$ with a single object $\bullet$ is simply a category $\mathcal{D} := \mathrm{Hom}(\bullet, \bullet)$ equipped with a bifunctor $* : \mathcal{D} \times \mathcal{D} \to \mathcal{D}$.



If $\mathcal{C}$ is strict $(\alpha, \ell, \rho = \mathrm{id})$ then $*$ is a unital associative product on $\mathcal{D}$ and the pair $(\mathcal{D}, *)$ is called a **strict monoidal category**. In the general case, the quintuple $(\mathcal{D}, *, \alpha, \ell, \rho)$ is called a *monoidal category*.

A bicategory with one object is a monoidal category.

Examples of monoidal categories

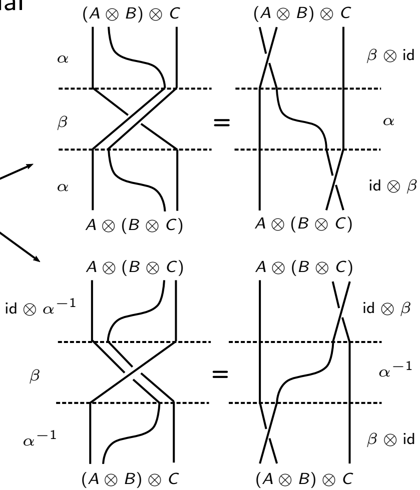
- $(\mathrm{Cat}, \times)$ of (small) categories and functors
- $(\mathsf{Vect}_{\mathbb{R}}, \otimes_{\mathbb{R}})$ of real vector spaces and linear transformations
- $(\mathsf{Rep(G)}, \otimes)$ of finite dim. $\mathbb{C}$-reps. of group G and intertwiners
- $(\mathsf{Cob}_n, \amalg)$ of $(n-1)$-dim. closed oriented manifolds and cobordisms

A (topological) structure on a monoidal category

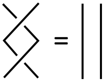
A *braided monoidal category* is a monoidal category $(\mathcal{C}, \otimes, \alpha, \ell, \rho, \mathbb{1})$ equipped with a braiding $\{\beta : A \otimes B \to B \otimes A\}_{A,B \in \mathcal{C}}$

$\beta :=$ 

which satisfies the conditions



A *symmetric monoidal category* is a braided monoidal category such that $\beta^2 = \text{id} : A \otimes B \to A \otimes B$.



Braided and symmetric monoidal categories

Braided and symmetric monoidal categories are useful in

- obtaining knot and link invariants,
- axiomatization of topological quantum field theories,
- formulation of topological quantum computation,
- classification of topological phases of matter,
- representations of groups or Hopf algebras,
- ...

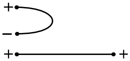
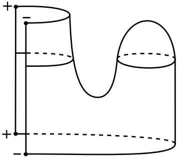
Are there similar structures on bicategories?

In the case of bicategories there is an additional monoidal bicategory between braided and symmetric called *sylleptic monoidal bicategory*.

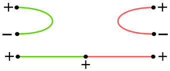
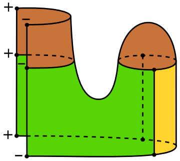
- Monoidal bicategories
- Braided monoidal bicategories
- Sylleptic monoidal bicategories
- Symmetric monoidal bicategories

We skip definitions of these bicategories. Instead, we give two examples of symmetric monoidal bicategories .

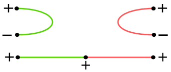
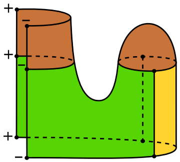
Examples of monoidal bicategories and extended TFTs

	$(\mathrm{Bord}_2, \amalg)$	$(\mathrm{Alg}^2_{\mathbb{k}}, \otimes_{\mathbb{k}})$
Objects:		$\mathbb{k}$-algebras
1-morphisms:		Bimodules
2-morphisms:		Bimodule maps

Examples of monoidal bicategories and extended TFTs

	$(\text{Bord}_2, \amalg)$	$(\text{Alg}^2_{\mathbb{k}}, \otimes_{\mathbb{k}})$
Objects:	$\overset{+}{\bullet}$ $\overset{-}{\bullet}$	$\mathbb{k}$-algebras
1-morphisms:		${}_A M \otimes_A N_A$
2-morphisms:		${}_A R \otimes_A S_A \to {}_A M \otimes_A N_A \to {}_A T_A$

Examples of monoidal bicategories and extended TFTs

	$(\mathrm{Bord}_2, \amalg)$	$(\mathrm{Alg}^2_{\Bbbk}, \otimes_{\Bbbk})$
Objects:	$\overset{+}{\bullet}$ $\overset{-}{\bullet}$	$\Bbbk$-algebras
1-morphisms:		${}_A M \otimes_A N_A$
2-morphisms:		${}_A R \otimes_A S_A \to {}_A M \otimes_A N_A \to {}_A T_A$

Definition (Schommer-Pries)

A two-dimensional extended TFT is a symmetric monoidal 2-functor

$$Z : (\mathrm{Bord}_2, \amalg) \to (\mathrm{Alg}^2_{\Bbbk}, \otimes_{\Bbbk}).$$

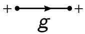
Two-dimensional extended HFTs

Definition (S.)

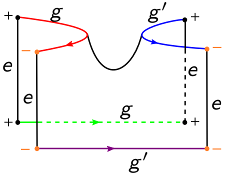
A two-dimensional extended HFT with target $X \simeq K(G,1)$ is a symmetric monoidal 2-functor $Z : (X\mathrm{Bord}_2, \amalg) \to (\mathrm{Alg}^2_{\Bbbk}, \otimes_{\Bbbk})$.

$$X\mathrm{Bord}_2 \xrightarrow{\quad Z \quad} \mathrm{Alg}^2_{\Bbbk}$$

$A \otimes_{\Bbbk} B$ — $\Bbbk$-algebra



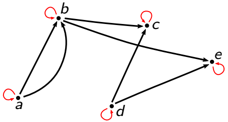
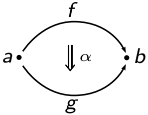
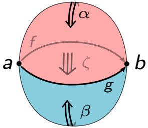
${}_A(M_g)_A$ — (A, A)-bimodule



$${}_{A\otimes_{\Bbbk}B}R_g \otimes_{\Bbbk} (S_{g'})_{A\otimes_{\Bbbk}B}$$
$$\Big\downarrow$$
$${}_A(M_g)_A \otimes_{\Bbbk} {}_B(N_{g'})_B$$

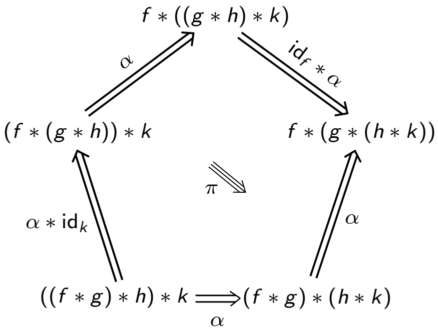
$(A \otimes B, A \otimes B)$-bimodule map

What's next after 2-categories?

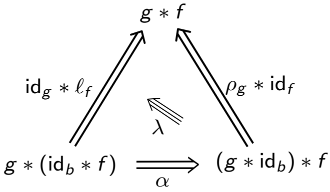
3-category $\mathcal{C}$	Visualization
⊛ Objects ⊛ 1-morphisms	
⊛ 2-morphism $\alpha : f \to g$	
⊛ 3-morphism $\zeta : \alpha \to \beta$	

Descriptive definition of tricategory

There are invertible modifications subject to conditions given by a list of commutative diagrams (see Gordon-Power-Street).



⊛ For any two objects $a, b \in \mathcal{C}$ $\mathrm{Hom}_{\mathcal{C}}(a, b)$ is a bicategory.



⊛ A tricategory with one object is a monoidal bicategory.

⊛ A tricategory with only one object and one 1-morphism is a braided monoidal category.

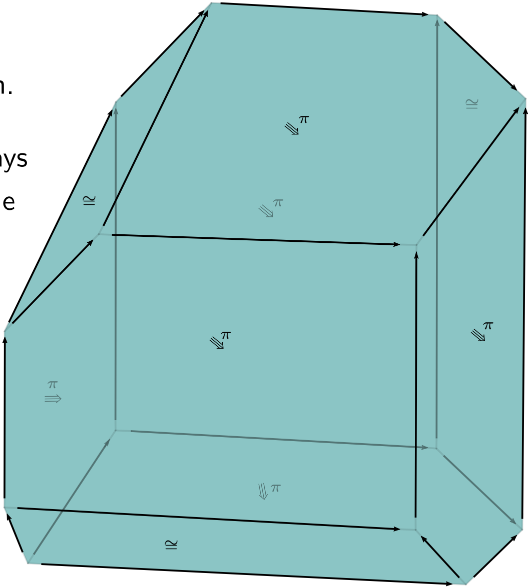
Descriptive definition of tricategory: Associahedron K_5

⊛ One of the conditions is given by the associahedron.

⊛ Corners are all different ways of composing 5 composable 1-morphisms.

⊛ # of vertices (K_{n+1}) || n-th Catalan number

⊛ Edges are associator 2-morphisms.



Descriptive definition of n-category

An n-category $\mathcal{C}$ consists of

- Objects,
- 1-morphisms between objects,
- 2-morphisms between 1-morphisms,

$$\vdots \qquad \vdots$$

- $(n-1)$-morphisms between $(n-2)$-morphisms,
- n-morphisms between $(n-1)$-morphisms.

Definition

A *strict n-category* is a diagram of sets with $s \circ s = s \circ t$ and $t \circ s = t \circ t$.

$$C_n \underset{t_n}{\overset{s_n}{\rightrightarrows}} C_{n-1} \underset{t_{n-1}}{\overset{s_{n-1}}{\rightrightarrows}} \cdots \underset{t_2}{\overset{s_2}{\rightrightarrows}} C_1 \underset{t_1}{\overset{s_1}{\rightrightarrows}} C_0 \quad + \text{assoc} + \text{unit conds.}$$

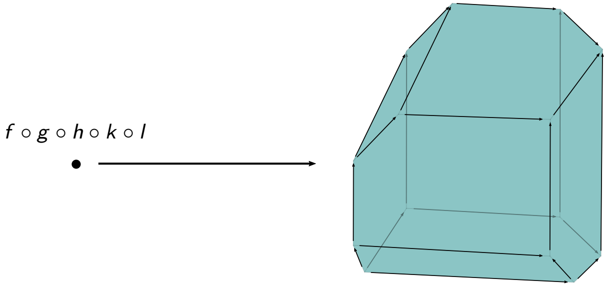
Definition

Let 0-category be a set. Then a *strict n-category* (all coherence isomorphisms = identities) is a category enriched over $(n-1)$-Cat.

Weak n-categories

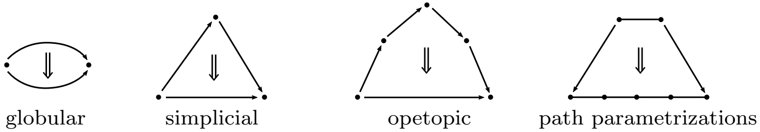
In a **strict** n**-category***, we can take a pasting diagram of cells and compose them in any order we like, the result will be the same. To make a* **weak** n**-category***, we are going to "strectch out" a strict n-category a bit, so that there is a bit of "distance" in between these composites done in different orders but not too much. And it shouldn't be empty space in between; they must be connected via mediating cell.*

Eugenia Cheng and Aaron Lauda



On different definitions of weak n-category

Authors	**Shapes of cells**
Batanin	globular
Baez-Dolan, Cheng, et al.	opetopic
Joyal	globular/simplicial
Leinster	globular
May	-
Penon	globular
Simpson	simplicial/globular
Street	simplicial
Tamsamani	simplicial/globular
Trimble	path parametrizations



The main principle of higher category theory

The main principle of higher category is not to say things are equal but only isomorphic.

$$\text{equality} \rightsquigarrow \text{isomorphism}$$

Why? Recall definitions

- 1-category: $(h \circ g) \circ f = h \circ (g \circ f)$,
- 2-category: $\alpha_{h,g,f} : (h \circ g) \circ f \xrightarrow{\simeq} h \circ (g \circ f)$,
- 3-category: $\pi_{k,h,g,f} : (\mathrm{id} * \alpha) \circ \alpha \circ (\alpha * \mathrm{id}) \xrightarrow{\simeq} \alpha \circ \alpha$,

$\vdots \qquad \vdots \qquad \vdots$

Every n-category can be regarded as an $(n+1)$-category with only identity $(n+1)$-morphisms i.e. equalities.

Thus, every n-category is indeed an ∞-category whose j-morphisms for $j > n$ are identity morphisms.

Grothendieck's dream: "The Homotopy Hypothesis"

Definition

A space X is called *n-type* if $\pi_k(X) = 0$ for all $k > n$.

Example

S^1, T^2, Σ_g are 1-types and $\mathbb{CP}^\infty$ is a 2-type.

The Homotopy Hypothesis

The study of n-groupoids is the same as the study of n-types.

Fundamental n-groupoid $\pi_{\leq n}(X)$ of a topological space X has

- points of X as objects,
- paths between points as 1-morphisms,
- paths between paths i.e. homotopies as 2-morphisms,

$\vdots \qquad \vdots \qquad \vdots$

- homotopy classes of $(n-1)$-fold homotopies as n-morphisms.

The Homotopy Hypothesis

Note that in $\pi_{\leq n}X$, composition is not associative, nor unital, nor the morphisms are invertible on the nose; **only up to homotopy**.

The Homotopy Hypothesis

There is an equivalence of $(n+1)$-categories $n\text{Types} \underset{|-|_n}{\overset{\pi_{\leq n}}{\rightleftarrows}} n\text{Grpds}$

where $|-|_n$ is the (geometric) realization functor and nTypes has

- n-types as objects,
- continuous maps as 1-morphisms,
- homotopies as 2-morphisms,
- homotopies between homotopies as 3-morphisms,

$\vdots \qquad \vdots \qquad \vdots$

- homotopy classes of n-fold homotopies as $(n+1)$-morphisms.

The homotopy hypothesis in the limit

In the limit $n \to \infty$ we obtain all spaces (CW complexes).

The Homotopy Hypothesis for $n = \infty$

There is an equivalence of ∞-categories $\mathit{Spaces} \underset{|-|_\infty}{\overset{\pi_\infty}{\rightleftarrows}} \infty\text{Grpds}$ where Spaces is the ∞-category of spaces (CW complexes).

Considering Kan complexes as a model for ∞-groupoids we can take π_∞ as the singular simplicial set functor Sing. Then, the Quillen adjunction $\text{Top} \underset{|-|_\infty}{\overset{\pi_\infty = \text{Sing}}{\rightleftarrows}} \text{SSets}$ is the equivalence of ∞-categories.

Challenging question: *Find a definition of n-groupoids so that the Homotopy hypothesis holds in both finite and infinite cases.*

There are various different attempts/answers; see nLab and Mathoverflow

Strict n-groupoids cannot model n-types

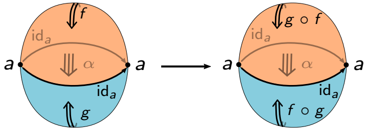
Theorem (Simpson)

Strict 3-groupoids are not sufficient to model all 3-types.

Example (Simpson, Berger)

One cannot obtain the 3-type of S^2 from a realization functor defined on strict 3-groupoids.

Consider a 3-groupoid with a single object **a** and a single 1-morphism id_a.



In a strict 3-groupoid α is identity.

In a weak 3-groupoid α is braiding.

⊛ There exist weak 3-groupoids not equivalent to strict 3-groupoids.

Suspension and Stabilization

	Category	Bicategory	Tricategory	Tetracategory
One object	✓	✓	✓	✓
One 1-morphism			✓	✓
One 2-morphism				✓
	Monoid	Monoidal category	Braided monoidal category	Symmetric monoidal category

Definition (Baez-Dolan)

An m-tuply monoidal n-category is an $(m+n)$-category with only one i-morphism for $i < m$.

The Periodic Table

	$n=0$	$n=1$	$n=2$
$m=0$	Set	Category	2-category
$m=1$	Monoid	Monoidal category	Monoidal bicategory
$m=2$	Commutative monoid	Braided mon category	Braided mon bicategory
$m=3$	"	Symmetric mon category	Sylleptic mon bicategory
$m=4$	"	"	Symmetric mon bicategory
$m=5$	"	"	"

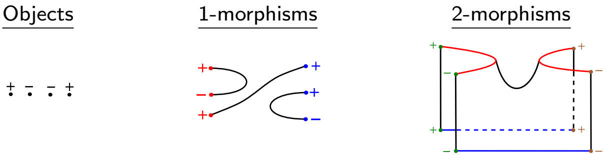
Stabilization Hypothesis (Baez-Dolan)

Periodic table stabilizes for $m \geq n+2$ just as $\pi_{m+n}(S^m)$.

There are different approaches and proofs for this conjecture; see ▸ nLab

The n-category of cobordisms

Recall the extended 2-dimensional bordism bicategory Bord_2



Extending Bord_2 inductively we obtain the n-category Bord_n which has

- compact oriented 0-manifolds as objects,
- oriented cobordisms as 1-morphisms,
- oriented cobordisms between those as 2-morphisms,

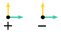
$\vdots \qquad \vdots \qquad \vdots$

- diffeomorphism classes of n-fold oriented cobordisms relative to boundary/corners as n-morphisms.

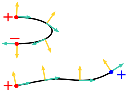
The Cobordism Hypothesis (Baez-Dolan)

Symmetric monoidal bicategory $(\mathrm{Bord}_2^{fr}, \amalg)$

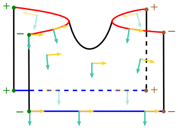
<u>Objects</u>



<u>1-morphisms</u>



<u>2-morphisms</u>



Definition

An n-**dimensional fully-extended framed topological field theory** is a symmetric monoidal n-functor $Z : \mathrm{Bord}_n^{fr} \to \mathcal{C}$ where $\mathcal{C}$ is any symmetric monoidal n-category.

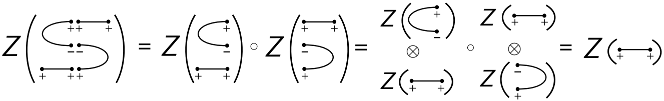
The Cobordism Hypothesis (Baez-Dolan)

Each n-dimensional fully-extended framed topological field theory is determined by the image of framed point which is a <u>fully-dualizable</u> object.

$\mathcal{C}$-valued n-dim fully-extended framed TFTs $\leftrightarrow$ Fully-dualizable objects in $\mathcal{C}$

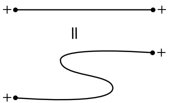
Why not any object but a fully-dualizable one?

Consider a 1-dimensional framed (oriented) TFT:

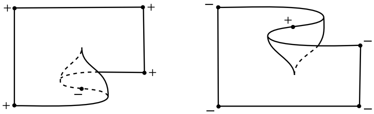


In Bord_2 (higher) duality data replaces equalities with isomorphisms

in Bord_1

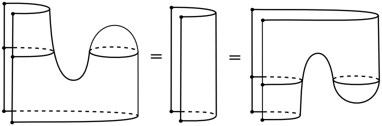


in Bord_2



Generators: handle attachments

Relations: handle cancellations



Lurie's reformulation

Extend the n-category Bord_n^{fr} to an (∞, n)-category Bord_n^{fr} by taking

- diffeomorphisms of n-manifolds with corners as $(n+1)$-morphisms,
- isotopies of such diffeomorphisms as $(n+2)$-morphisms,
- isotopies of these isotopies as $(n+3)$-morphisms,

$\vdots \qquad \vdots \qquad \vdots$

Definition (Lurie)

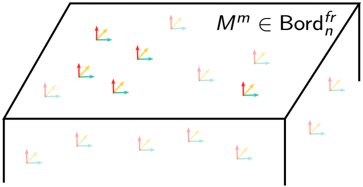
An n-**dimensional fully-extended framed topological field theory** is a symmetric monoidal (∞, n)-functor $Z : \mathrm{Bord}_n^{fr} \to \mathcal{C}$ where $\mathcal{C}$ is any symmetric monoidal (∞, n)-category.

The Cobordism Hypothesis (Lurie)

Let $\mathrm{Fun}^{\otimes}(\mathrm{Bord}_n^{\mathrm{fr}}, \mathcal{C})$ be the (∞, n)-category of $\mathcal{C}$-valued fully-extended framed TFTs. Then there is a weak homotopy equivalence of spaces

$$\mathrm{Fun}^{\otimes}(\mathrm{Bord}_n^{\mathrm{fr}}, \mathcal{C}) \simeq (\mathcal{C}^{fd})^{\sim}.$$

Structured Cobordism Hypothesis



$$T(M \times (-\varepsilon,\varepsilon)^{n-m}) \circlearrowleft O(n) \xrightarrow{\;Z\;} O(n) \circlearrowleft \text{duality data } Z(M) \text{ belongs}$$

$$O(n) \circlearrowleft \mathrm{Bord}_n^{fr} \rightsquigarrow O(n) \circlearrowleft (\mathcal{C}^{fd})^{\sim}$$

A group homomorphims $\Gamma \to O(n)$ leads to a Γ-structured (∞, n)-bordism category Bord_n^{Γ} of manifolds with Γ-tangential structure.

Lurie's Γ-Structured Cobordism Hypothesis

There is a weak homotopy equivalence $\mathrm{Fun}^{\otimes}(\mathrm{Bord}_n^{\Gamma}, \mathcal{C}) \simeq ((\mathcal{C}^{fd})^{\sim})^{h\Gamma}$ of spaces where $((\mathcal{C}^{fd})^{\sim})^{h\Gamma} = \mathrm{Hom}_{\Gamma}(E\Gamma, (\mathcal{C}^{fd})^{\sim})$ is the space of homotopy Γ-fixed points.

- $\Gamma = \{e\}$ corresponds to framed TFTs i.e. $\mathrm{Bord}_n^{\{e\}} = \mathrm{Bord}_n^{fr}$,
- $\Gamma = SO(n)$ corresponds to oriented TFTs i.e. $\mathrm{Bord}_n^{SO(n)} = \mathrm{Bord}_n$,

Verifying $(G \times SO(2))$-structured Cobordism Hypothesis

- For any discrete group G, $\Gamma = G \times SO(2)$ corresponds to certain oriented 2-dimensional extended homotopy field theories,
- When $\mathbb{k} = \overline{\mathbb{k}}$ and $\mathrm{char}(\mathbb{k}) = 0$, comparing the classification of $\mathrm{Alg}^2_{\mathbb{k}}$-valued E-HFT and Davidovich's homotopy $(G \times SO(2))$-fixed point computations verifies this hypothesis for $\mathrm{Alg}^2_{\mathbb{k}}$-valued $(G \times SO(2))$-structured E-TFTs.

Verifying $(G \times SO(2))$-structured Cobordism Hypothesis

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Thanks for taking the time to read!