Math 3GR3, Tutorial 3

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Topics: $\mathbb{Z}_n$. Group of units. Subgroups. Cyclic groups.

Question 1 (Judson 3.5.17)**.** Give an example of three different groups with 8 elements. Why are the groups different?

Aside. Much of abstract algebra in the 20th century was devoted to the "classification problem", determining exactly how many unique groups with n elements there are, for each n. In the case of finite *simple* groups, this was solved in ∼2004, culminating the work of around 100 authors spanning half a century.

The takeaway is this: questions like the previous question are very difficult in general. Later in the course, we will learn what it means for a group to be simple, and the notion of *isomorphic* groups, that is, when are two groups "the same".

Question 2 (Judson 3.5.47)**.** Prove or disprove: If H and K are subgroups of a group G, then $HK \coloneqq \{hk \mid h \in H \text{ and } k \in K\}$ is a subgroup of G. What if G is abelian?

Question 3 (Judson 3.5.52)**.** Prove or disprove: Every proper subgroup of a nonabelian group is also nonabelian.

Question 4 (Judson 4.5.26)**.** Prove that $\mathbb{Z}_p$ has no nontrivial subgroups if p is prime.

Question 5 (Judson 4.5.30)**.** Suppose that G is a group and let $a, b \in G$. Prove that if $|a| = m$ and $|b| = n$, with $\gcd(m, n) = 1$, then $\langle a\rangle \cap \langle b\rangle = \{e\}$.

Question 6 (Judson 4.5.34)**.** Let G be an abelian group of order pq where $\gcd(p, q) = 1$. If G contains elements a and b of order p and q respectively, then show that G is cyclic.