

Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 9/Week 10

Topics: Multivariable functions. Limits and continuity.

1. Find and sketch the domains of each function.

(a) $f(x, y) = \ln(x + y + 1)$

(b) $g(x, y) = \sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2}$

2. The *temperature-humidity index (humidex)* I is the perceived air temperature when the actual temperature is T and the relative humidity is h . That is, $I(T, h)$ is a function of two variables. Consider the following table of values of I for varying T and h .¹ Note that the humidity is measured as a percentage, and the temperatures are in Fahrenheit.

$T \setminus h$	20	30	40	50	60	70
80	77	78	79	81	82	83
85	82	84	86	88	90	93
90	87	90	93	96	100	106
95	93	96	101	107	114	124
100	99	104	110	120	132	144

(a) What is the value of $I(95, 70)$. What does it represent?

(b) For what value of T is $I(T, 50) = 96$?

(c) What is the meaning of the function $I(100, h)$? Express your answer in terms of the trace.

3. Find the limits, if they exist.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + 2y^2)}{3x^2 + y^2}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^4 + y^4}$

4. Find the limits, if they exist.

¹Source: National Oceanic & Atmospheric Administration (NOAA).

(a) $\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2)$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{e^{-x^2-y^2} - 1}{x^2 + y^2}$

5. Let $f(x, y) = \frac{x^3 + y^3}{x^2 + y^2}$. We will show that $f(x, y) \rightarrow 0$ as (x, y) approaches the origin.

(a) Show that

$$|x^3| \leq |x|(x^2 + y^2) \quad \text{and} \quad |y^3| \leq |y|(x^2 + y^2)$$

(b) Using part (a) and some other properties of the absolute value, show that $|f(x, y)| \leq |x| + |y|$.

(c) Use part (b) and the Squeeze Theorem to show that $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 0$.