Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 5/Week 6

Topics: Power series. Taylor series.

Note: Solutions to these problems will be posted on Avenue. At the end of the week, see Content → Tutorials → Problems with Solutions.

1. Evaluate the limit without l'Hôpital's rule.

$$\lim_{x \to 0} \frac{\sin x - x}{x^3}$$

2. Find the antiderivative.

$$\int \frac{e^x}{x}\, dx.$$

3. Suppose that the (Taylor) series $\sum_{n=0}^{\infty} c_n(x-2)^n$ converges at $x = -2$ and diverges at $x = 8$. Do each of the following series converge, diverge, or is there not enough information to conclude? Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty} c_n$ (b) $\displaystyle\sum_{n=0}^{\infty} c_n 7^n$ (c) $\displaystyle\sum_{n=0}^{\infty} c_n(-5)^n$ (d) $\displaystyle\sum_{n=0}^{\infty} c_n 4^n$

4. Let m_0 be the mass of an object at rest and let c be the speed of light. Einstein's theory of special relativity says that the mass of the object when it is moving at velocity v is given by

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}.$$

The kinetic energy of the object is the difference between its total energy and its energy at rest, $K = mc^2 - m_0c^2$. Show that when v is very small (compared to c), the formula for $m(v)$ agrees with $K \approx \frac{1}{2}m_0v^2$, from classical physics.

[Hint: you may find it useful to recall the result of Theorem 3 in Section 10.8. That is, for any exponent a and any number x with $|x| < 1$, we have $(1+x)^a = \sum_{n=0}^{\infty} \binom{a}{n} x^n$. Also, use the definition of the choose function that says $\binom{a}{n} = a(a-1)\cdots(a-n+1)/n!$ which is defined for all real values of a.]