Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 2/Week 3

Topics: Differential equations. Logistic equation. First-order linear differential equations.

Note: Solutions to this problem set will be posted on Avenue. See Content → Tutorials → Problems with Solutions at the end of the week.

1. True or false?

   (a) The differential equation is $e^x y' = y$ is linear.

   (b) The differential equation $y' + xy = e^y$ is linear.

2. Which method would you use to solve each of the following differential equations?

   (a) $y' = xe^{-\sin x} - y\cos x$

   (b) $\frac{dx}{dt} = 1 - t + x - tx$

   (c) $2ye^{y^2}y' = 2x + 3\sqrt{x}$

   (d) $x^2y' - y = 2x^3e^{-1/x}$

3. A remote island is measured to have an initial dragon population of 200. A year later the population is 350.

   (a) Let $P(t)$ be the dragon population at time t, where t is in years. Assuming the island has a carrying capacity of 500 dragons, use the logistic equation to model the dragon population and solve for P.

   (b) Sketch a graph of your model from part (a).

   (c) How quickly is the dragon population increasing when the population is 300?

4. A stream feeds into a lake at a rate of 1000 L/day. The stream is polluted with a toxin whose concentration is 20 g/L. Assume that the lake has volume 10^6 L and that water flows out of the lake at the same rate of 1000L/day.

   (a) Find the equation $s(t)$ for the amount of toxin in the lake, assuming $s(0) = 0$.

   (b) Find the equation $c(t)$ for the concentration of toxin in the lake, assuming $c(0) = 0$.

5. **Bonus** (time permitting). A *Bernoulli differential equation* is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)y^n.$$

Observe that if $n = 0$ or $n = 1$ then the equation is linear.

(a) Show that for other values of n, the substitution $u = y^{1-n}$ transforms the Bernoulli equation into the linear equation

$$\frac{du}{dx} + (1-n)P(x)u = (1-n)Q(x).$$

(b) Solve the differential equation

$$xy' + y = -xy^2.$$