Math 1XX3 Tutorial Problems

for T04, T07 with Mike

Tutorial 10/Week 11

Topics: Partial derivatives. Differentiability.

1. (a) What is the mistake in the following computation? $\frac{\partial}{\partial x}(x^2y^2) = x^2(2y) + y^2(2x)$

   (b) Which of the following partial derivatives should be evaluated *without* the quotient rule? Calculate the partial derivative.

   i. $\frac{\partial}{\partial x}\frac{xy}{y^2+1}$ ii. $\frac{\partial}{\partial y}\frac{xy}{y^2+1}$ iii. $\frac{\partial}{\partial x}\frac{y^2}{y^2+1}$

2. Find $f(x, y)$ such that

$$\begin{cases} f_x = \sin y + \dfrac{1}{1-xy} \\ f(1, y) = \sin y \end{cases}$$

3. Find $\frac{\partial z}{\partial x}$ for the surface $e^z = xyz$. *[Hint: implicit differentiation]*

4. (a) Use Clairaut's Theorem to show that if the third-order derivatives of $f(x, y)$ are continuous, then

$$f_{xyy} = f_{yxy} = f_{yyx}.$$

   (b) Assuming that the assumptions of Clairaut's Theorem hold, which of the following partial derivatives are equal to f_{xxy}?

   i. f_{xyx} ii. f_{yyx} iii. f_{xyy} iv. f_{yxx}

5. Prove that there does not exist any function $f(x, y)$ such that $f_x = xy$ and $f_y = x^2$.