Math 701, Midterm Test

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Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

All rings on this test have a 1.

1. Show that all one-generated or cyclic left R-modules are isomorphic to one of the form R/I for some left ideal $I \subset R$.

2. Show that if R is an integral domain then for any left R-module M,

$$\{m \in M : rm = 0 \text{ for some } r \neq 0, r \in R\}$$

is a submodule of M.

3. Show that if $e \in R$ has the property that $er = re$ for all $r \in R$ and $e^2 = e$ then for any left R-module M, $M = eM \oplus (1-e)M$.

4. Suppose that I is an ideal of R and N is a left R-module. Remember that IN is the submodule of N generated by the elements rn for $r \in I$ and $n \in N$. Show that

$$R/I \otimes_R N \cong N/IN.$$

5. (a) Define the terms projective module, injective module and, for commutative rings, flat modules.

   (b) Show that projective modules are flat.