Assignment 5, Math 4L3
Due Dec. 5, in class

1. Show that every formula is equivalent to a quantifier-free formula modulo the theory of infinite sets in the language with just equality. Hint: this follows the outline of our proof of quantifier elimination for dense linear orders without endpoints.

2. Prove the theorem that I called the tautology theorem in class (Goldrei, Thm. 5.3). Goldrei gives some hints.

3. Prove that there is only one countable equivalence relation with infinitely many classes, all of which are infinite, up to isomorphism. Conclude from this that any two equivalence relations with infinitely many classes, all infinite, are elementarily equivalent. Hint: reason as we did with dense linear orders without endpoints.

4. Show that there is a countable ordered field, elementarily equivalent to the real numbers, which contains an element that is greater than N for all N. In particular, conclude that it is not a subfield of the reals.