Assignment 2, Math 4GR3

Due Feb. 14, uploaded to Avenue (Happy Valentines Day!)

1. Prove the following version of the second isomorphism theorem: Suppose G is a group with two normal subgroups H and K. Then

$$HK/K \cong H/(H \cap K).$$

   Notice that it is enough that H normalizes K; that is, for every $h \in H$, $hKh^{-1} = K$.

2. (The third isomorphism theorem) Suppose that A and N are normal subgroups of G and that $A \subset N$. Prove that

$$(G/A)/(N/A) \cong G/N.$$

3. Questions from Judson: Chap. 13, # 11, Chap. 14, # 4, 20, 23

4. (Bonus question from Dr. Cousins) The Schroeder-Berstein theorem says that if X and Y are two sets and there is an injection from X to Y and also an injection from Y to X then there is a bijection between X and Y. Prove or disprove the same thing for groups. That is, if G and H are groups such that $g\colon G \to H$ is an injective group homomorphism and $h\colon H \to G$ is also an injective group homomorphism then G and H are isomorphic.