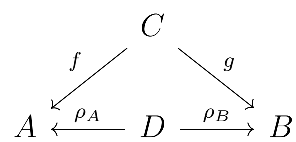
Assignment 1, Math 4GR3
Due Jan. 24, uploaded to Avenue

1. Suppose that A and B are two groups. In class we discussed the abstract notion of the product of two groups. That is, D together with two group homomorphisms $\rho_A\colon D \to A$ and $\rho_B\colon D \to B$ is a product of A and B if whenever C is a group and f and g are group homomorphisms are as pictured:



then there is a unique group homomorphism $h\colon C \to D$ such that $\rho_A \circ h = f$ and $\rho_B \circ h = g$. Show that if D', ρ'_A and ρ'_B is also a product of A and B then there is a unique isomorphism $i\colon D \to D'$ such that $\rho_A = \rho'_A \circ i$ and $\rho_B = \rho'_B \circ i$.

2. Show that if we have groups G_i for $i \leq n$ and normal subgroups N_i of G_i for $i \leq n$ then

$$G_1/N_1 \times G_2/N_2 \times \ldots G_n/N_n \cong (G_1 \times \ldots \times G_n)/(N_1 \times \ldots \times N_n).$$

3. Prove that there is only one free abelian group up to isomorphism on n generators. That is, if F is a free abelian group on generators $x_1, \ldots, x_n$ and G is a free abelian group on generators $y_1, \ldots, y_n$ then there is a unique isomorphism $f\colon F \to G$ such that $f(x_i) = y_i$ for $i \leq n$.

4. Show that if A and B are abelian groups, $\varphi_i\colon A \to B$ are group homomorphisms for $i \leq n$ and $m_1, \ldots, m_n \in Z$ then

$$m_1\varphi_1 + \ldots + m_n\varphi_n$$

is a group homomorphism from A to B.