Assignment 4, Math 2S3
Due Mar. 11 in class

(1) In Chapter XI of Lang's book, the first section is entitled "The Euclidean Algorithm" but the main theorem he proves is really just long division; let's actually prove the Euclidean algorithm: suppose that $f, g \in K[x]$. Use long division to write $g = qf + r$ with $deg(r) < deg(f)$. If $r \neq 0$, show that the greatest common divisor of g and f is the same as the greatest common divisor of f and r. Use this result to describe an algorithm to compute the greatest common divisor of f and g.
(2) Suppose that A is a complex $n \times n$ matrix with characteristic polynomial
$$\prod_{i=1}^{m}(x - \lambda_i)^{k_i}$$
and $f(x)$ is a complex polynomial. What is the characteristic polynomial of $f(A)$?
(3) How many 6×6 complex matrices are there with the characteristic polynomial x^6 up to similarity?
(4) In class we developed the formula for the determinant of an $n \times n$ matrix A with entries a_{ij}
$$\det(A) = \sum_{1 \leq i_1, \ldots, i_n \leq n} (-1)^{N(i_1, \ldots, i_n)} a_{1i_1} \ldots a_{ni_n}$$
where the $i_1, \ldots, i_n$ in the summation are all distinct and $N(i_1, \ldots, i_n)$ is the inversion number for this permutation. Show that this formula satisfies the conditions we set for determinants; that is, show that it is multilinear, alternating and $\det(I) = 1$.