Math 2S3, Test 2
March 13, 2015

Please write complete answers to all of the questions in the test booklet provided. Partial credit may be given for your work. Unless otherwise noted, you need to justify your solutions in order to receive full credit. Please be sure to include your name and student number on all sheets of paper that you hand in.

1. (5 marks) Suppose that $f, g \in K[x]$ and A is an $n \times n$ matrix over K. Explain why $(fg)(A) = f(A)g(A)$.

2. (5 marks)

   (a) Give the definition of minimal polynomial for an $n \times n$ matrix over K.

   (b) If A is an $n \times n$ matrix over K, show that the minimal polynomial of A divides the characteristic polynomial of A.

3. (5 marks)

   (a) Suppose that A is an $n \times n$ matrix over K and W is a subspace of K^n. Define what it means to say that W is A-invariant or invariant for A.

   (b) Show that if $f \in K[x]$ and A is an $n \times n$ matrix over A then the kernel of $f(A)$ is A-invariant.

4. (5 marks) Up to similarity, how many matrices have the characteristic polynomial $(x-1)(x-2)^2(x-3)^3$?

5. (5 marks)

   (a) Suppose that A is a single Jordan block of size $n \times n$ with eigenvalue λ. What is the dimension of the λ-eigenspace for A?

   (b) Show that if A and B are two similar $n \times n$ complex matrices then for any λ, the dimension of the λ-eigenspace for A equals the dimension of the λ-eigenspace for B.

   (c) For an $n \times n$ complex matrix A, what is the relationship between the dimension of the λ-eigenspace for A and the number of Jordan blocks with eigenvalue λ in the Jordan canonical form of A?