Mathematics 2R3 Practice Test 1

Last Name:____________________________ Initials:__________

Student No.:____________________________

- The test is 50 minutes long.
- The test has 6 pages and 5 questions and is printed on BOTH sides of the paper.
- You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancies to the attention of the invigilator.
- Attempt all questions and write your answers in the space provided.
- Marks are indicated next to each question; the total number of marks is 25.
- You may use a Casio fx-991 calculator (no communication capability); no other aids are not permitted.
- Use pen to write your test. If you use a pencil, your test will not be accepted for regrading (if needed).

Good Luck!

continued ...

1. (5 marks) Put your answer in the space provided for each part.

   (a) For all complex numbers z, $|z| = |\bar{z}|$. True or false.

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   (b) $1, 1+x$ and $x^2 + x^3$ is a basis for the vector space of polynomials of degree less than or equal to 3. True or false.

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   (c) If $u = (3, 1+2i, 2)$ and $v = (i, 1-i, 0)$ in C^3 then $u \cdot v =$ __________

   (d) Suppose V is a real inner product space and $u, v \in V$ such that $\langle u, u\rangle = 2$, $\langle u, v\rangle = -1$ and $\langle v, v\rangle = 1$. Compute $||u + v||$.

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   (e) In the inner product space of continuous functions on $[-1, 1]$ with inner product given by

   $$\langle f, g\rangle = \int_{-1}^{1} fg dx,$$

   compute the inner product of 1 with x^2.

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continued . . .

2. (a) (2 marks) Express $\frac{i}{1+i}$ in the form $a + bi$.

(b) (3 marks) Find all complex numbers z such that $z^4 = -1$.

continued . . .

3. (5 marks) Show that the set of continuous real-valued functions f on $[0, 1]$ which satisfy $f(0) = f(1) = 0$ is a subspace of all continuous functions on $[0, 1]$.

continued ...

4. (5 marks) Suppose A is the invertible matrix

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{pmatrix}$$

We know that the function given by $\langle u, v \rangle = Au \cdot Av$ is an inner product on R^3. Compute the distance between $(1, 0, 0)$ and $(0, 1, 0)$ with respect to this inner product.

continued . . .

5. Let P_2 be the vector space of all polynomials of degree 2 or less. On P_2, define an inner product as follows: for $f, g \in P_2$

$$\langle f(x), g(x) \rangle = f(-1)g(-1) + f(0)g(0) + f(1)g(1).$$

Use the Gram-Schmidt process applied to the basis $\{1, x, x^2\}$ to produce an orthogonal basis for P_2 with respect to this inner product; you needn't normalize this basis.

THE END