Linear algebra, Math 2R3

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Sept. 4, 2019

Short-term outline

- Review of complex numbers, appendix B, supplementary material
- Review of real vector spaces, sections 4.1 – 4.3
- Introduce complex vector spaces, section 5.3

The complex numbers

- Introduce a new quantity, i, such that $i^2 = -1$.
- The complex numbers are then all expressions of the form $a + bi$ where a and b are real numbers.

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Operations on the complex numbers

- Addition:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

- Multiplication:

$$(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$$

Multiplicative inverse

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The conjugate

- If $z = a + bi$ then $\bar{z}$, the conjugate of z, is $a - bi$.
- Notice that $z\bar{z} = a^2 + b^2$ so

$$\frac{1}{z} = \frac{\bar{z}}{z\bar{z}}$$

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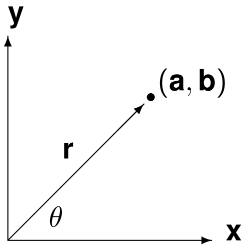
- The rational numbers, the real numbers and the complex numbers all obey the same algebraic rules e.g. commutativity and associativity of addition and multiplication, distributivity, and all non-zero elements have a multiplicative inverse. They are fields.
- The complex numbers have the property that if $p(x)$ is a non-zero polynomial with complex coefficients then p has a complex root - the complex numbers form an algebraically closed field.

How to picture complex numbers: the complex plane



- Associate to the complex number $z = a + bi$ the point on the plane (a, b). $r = \sqrt{a^2 + b^2}$ is called the modulus of z and written $|z|$.
- We saw that $z \cdot \bar{z} = |z|^2$.
- θ is called an argument for $a + bi$ and is only determined up to multiples of 2π.
- $a = r\cos(\theta)$ and $b = r\sin(\theta)$ so $z = r(\cos(\theta) + i\sin(\theta))$.