A fundamental problem

Problem

Given an $m \times n$ matrix A, find all the b's such that $Ax = b$ has a solution.

Diagonal matrices

Definition

For a square matrix $D = (d_{ij})$ is said to be diagonal if $d_{ij} = 0$ whenever $i \neq j$.

A diagonal matrix is invertible iff all its diagonal entries are non-zero.

Triangular matrices

Definition

Suppose that $A = (a_{ij})$ is an square matrix.

- A is said to be upper triangular if $a_{ij} = 0$ when $i > j$ i.e. when the entry is below the diagonal.
- A is said to be lower triangular if $a_{ij} = 0$ when $i < j$ i.e. when the entry is above the diagonal.

Triangular matrices, cont'd

Theorem (1.7.1)

- *The transpose of an upper triangular matrix is lower triangular and vice versa.*
- *The product of upper triangular matrices is upper triangular and the same for lower triangular matrices.*
- *A triangular matrix is invertible iff all its diagonal entries are non-zero.*
- *The inverse of an upper triangular matrix is upper triangular and the same for lower triangular matrices.*

Symmetric matrices

Definition

For a square matrix $A = (a_{ij})$ is said to be symmetric if $a_{ij} = a_{ji}$ for all i and j.

Theorem (1.7.2, 1.7.3 and 1.7.4)

Suppose that A and B are symmetric matrice. Then

- A^T *is symmetric;*
- $A + B$ *and* $A - B$ *are symmetric;*
- kA *is symmetric for all numbers* k*;*
- AB *is symmetric iff* $AB = BA$*; and*
- *If A is invertible then* A^{-1} *is also symmetric.*

Theorem (1.7.5)

If A is invertible then $A^T A$ *and* AA^T *are invertible.*