

Main facts about elementary matrices

Theorem

If E is an elementary matrix and EA makes sense then if $EA = B$, B is the matrix obtained from A by applying the elementary row operation associated with E .

Corollary

All elementary matrices are invertible.

An algorithm

Remember that A is invertible iff it is the product of elementary matrices. In fact, those elementary matrices correspond to the elementary row operations needed to row reduce A to reduced row echelon form.

Inverse algorithm

To find the inverse of an invertible matrix A , find a sequence of elementary row operations that reduces A to the identity and perform the same operations on the identity to produce A^{-1} .

Theorem

- *The linear system $Ax = b$ has either no solution, exactly one solution or infinitely many solutions.*
- *If A is invertible then $Ax = b$ has a unique solution.*

A fundamental problem

Problem

Given an $m \times n$ matrix A , find all the b 's such that $Ax = b$ has a solution.