Elementary matrices

Elementary row operations

The elementary row operations on a matrix are:

- Multiply a row by a non-zero constant
- Add a constant multiple of one row to another
- Interchange two rows

Definition

An elementary matrix is one obtained from an identity matrix by a single elementary row operation.

Theorem

If E is an elementary matrix and EA makes sense then if EA = B, B is the matrix obtained from A by applying the elementary row operation associated with E.

Corollary

All elementary matrices are invertible

An important theorem

Theorem (1.5.3)

The following are equivalent for a square matrix A:

1. *A is invertible.*
2. *The linear system $Ax = 0$ has only the trivial, 0, solution.*
3. *The reduced row echelon form of A is the identity matrix.*
4. *A is a product of elementary matrices.*

An algorithm

Inverse algorithm

To find the inverse of an invertible matrix *A*, find a sequence of elementary row operations that reduces *A* to the identity and perform the same operations on the identity to produce A^{-1}.