Another example, cont'd

- $$A^T = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 2 & 4 & 2 & 0 \\ -1 & -2 & -1 & 0 \\ 0 & -1 & -1 & 1 \\ 1 & 3 & 2 & -1 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

- $$rref(A^T) = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Diagonalization and eigenspaces

- Suppose that A is an $n \times n$ matrix and it has λ as an eigenvalue.
- Remember that we refer to all $x \in \mathbb{R}^n$ such that $Ax = \lambda x$ as the eigenspace corresponding to λ for the matrix A; it is a subspace of $\mathbb{R}^n$.
- We say that A is diagonalizable if there is an invertible matrix P such that $P^{-1}AP = D$ with D a diagonal matrix. In fact, the numbers on the diagonal of D are the eigenvalues of A and the columns of P are eigenvectors.
- It follows that if A is diagonalizable then there is a basis of $\mathbb{R}^n$ made up of eigenvectors of A.
- Conversely, if there is a basis for $\mathbb{R}^n$ made up of eigenvectors of $\mathbb{R}^n$ then A is diagonalizable.

Diagonalization and eigenspaces, cont'd

- Buried in here, there is an algorithm for determining whether a matrix is diagonalizable or not.
- Suppose that λ is an eigenvalue for A. We call the multiplicity of λ in the characteristic polynomial its algebraic multiplicity. We call the dimension of the eignespace corresponding to λ it geometric multiplicity.

Theorem

A is diagonalizable Iff for every eigenvalue λ of A, the algebraic multiplicity of λ equals the geometric multiplicity.

Where did the geometry go?

Definition

We call a basis S for $\mathbb{R}^n$ an orthogonal basis if for every distinct pair $u, v \in S$, $u \cdot v = 0$; we say that the basis is orthonormal if for every $u \in S$, $||u|| = 1$.

Fact

Any orthogonal set in $\mathbb{R}^n$ is linearly independent.

Coordinates with respect to an orthonormal basis

Suppose that $v_1, v_2, \ldots, v_n$ is an orthonormal basis for $\mathbb{R}^n$. If $v \in \mathbb{R}^n$ then

$$v = k_1 v_1 + \ldots k_n v_n$$

where $k_i = v \cdot v_i$ for $i = 1, \ldots, n$.

The Gram-Schmidt process

- Suppose we have a linearly independent set $u_1, \ldots, u_r$ which spans some subspace of $\mathbb{R}^n$. We would like to find an orthogonal basis for this same subspace.
- We construct an orthogonal set iteratively: let $v_1 = u_1$.
- Let $v_2 = u_2 - proj_{v_1}(u_2)$.
- Let $v_3 = u_3 - proj_{v_1}(u_3) - proj_{v_2}(u_3)$.
- In general, if we have already defined $v_1, \ldots, v_i$ then let

$$v_{i+1} = u_{i+1} - proj_{v_1}(u_{i+1}) - \ldots - proj_{v_i}(u_{i+1})$$

.

- Continue this process until you have dealt with all r vectors.
- The resulting $v_1, \ldots, v_r$ will be an orthogonal basis for W.

The Final Exam

- The final exam will be Dec. 18 at 4:30 pm and is 3 hours long; check the registrar's exam site for seating.
- The material covered for the final exam will be:
  - sections 1.1 - 1.8 and 2.1 - 2.3 (topics from the first test)
  - sections 5.1 - 5.2, 5.5, 3.1 - 3.5 and 10.1 - 10.3 from the 9th edition (topics from the second test)
  - sections 4.1 - 4.5, 4.7 - 4.8 and 6.3
- Weighting on the final will be 50% on the material since the last test; 25% on the material from each test.
- The exam will be multiple choice; bring an HB pencil. You may use a McMaster approved Casio fx-991 MS or MSPlus but no other aids. Bring your ID card with you to the exam.

The Final Exam, cont'd

- There are practice problems posted and a practice exam.
- Office hours Mon. Dec. 15 and Wed. Dec. 17, 10:30 - 12; by appointment otherwise. Matt will have office hours in the help centre; I will post them when I know them. The help centre is open every weekday from 2:30 - 6:30 during exams.
- There is a review session Tuesday, Dec. 16, 2:30 - 4:30 in ITB AB 102.