The cross product

Definition

Suppose that $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ are two vectors in $\mathbb{R}^3$. We define the cross product of u with v, written $u \times v$, as

$$(u_2 v_3 - v_2 u_3, v_1 u_3 - u_1 v_3, u_1 v_2 - v_1 u_2)$$

- $u \times v$ can also be described geometrically as a vector which is both orthogonal to u and v, has length $||u||||v|| \sin(\theta)$ where θ is the angle between u and v, and is oriented according to the right-hand rule (watch my hands).
- Theorems 3.5.1 and 3.5.2 have many properties of the cross product which follow immediately from one or the other way of looking at the cross product.

Area, volume and the determinant

- The area of the parallelogram determined by two vectors $u, v \in \mathbb{R}^3$ is $\|u \times v\|$.
- So if $u = (u_1, u_2)$ and $v = (v_1, v_2)$ then the area of the parallelogram determined by u and v is the absolute value of
$$\begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix}$$
- The area of the triangle determined by two vectors $u, v \in \mathbb{R}^3$ is $\|u \times v\|/2$. These formulas even work in $\mathbb{R}^2$.
- The volume of the parallelepiped determined by three vectors $u, v, w \in \mathbb{R}^3$ is $|u \cdot (v \times w)|$. So if A is the 3 x 3 matrix with rows given by u, v and w then the area of this parallelepiped is $|\det(A)|$.