

Definition

Suppose that A is a square matrix and for a number λ and non-zero column vector x , $Ax = \lambda x$. Then we call λ an eigenvalue for A and x an eigenvector.

Characteristic equation

For an $n \times n$ matrix A , the equation $\det(\lambda I - A) = 0$ where λ is a variable is called the characteristic equation for the matrix A . It is always a polynomial of degree n .

Definition

- We say that two square matrices A and B are similar if there is an invertible matrix P such that $A = P^{-1}BP$.
- We say that a matrix is diagonalizable if it is similar to a diagonal matrix.

Similarity properties

If two matrices are similar then they have the same determinant, trace, characteristic polynomial and eigenvalues.

Diagonalizability: a special case

Theorem

If A is $n \times n$ and the characteristic equation of A has n distinct roots then A is diagonalizable.

An algorithm for diagonalizing: a special case

If A is $n \times n$ and has n distinct eigenvalues $\lambda_1, \dots, \lambda_n$ then to find a diagonal matrix similar to A , do the following:

- for each $i = 1, \dots, n$, find a non-zero column vector v_i which is an eigenvector for λ_i i.e. find v_i such that $Av_i = \lambda_i v_i$.
- Let P be the matrix formed by placing v_i in the i^{th} column.
- Then P is invertible and $P^{-1}AP = D$ where D is the diagonal matrix with entry λ_i in the i^{th} place on the diagonal.

The test

- The first test is scheduled for Oct. 10 at 10:30 am (that is class time). The test will be 50 minutes.
- If your surname is in the range A - O, you write the test in T28/001; if your surname is in the range P - Z you write in T29/101.
- The test will be multiple choice; bring an HB pencil. McMaster approved Casio fx-991MS or MSplus calculators are allowed but no other aids.
- Please bring your ID card with you to the test.
- The test will cover sections 1.1 - 1.8 and 2.1 - 2.3.
- I will post supplementary questions and a practice test.
- Matt Luther will run a review session on Thursday, Oct. 9 from 5:30 - 7:30 in HH 302. Come prepared with questions and/or send him questions ahead of time.
- There will be no Friday morning tutorial on Oct. 10.