Some easy facts

- If A is a triangular matrix then $det(A)$ is the product of the diagonal entries.
- If a square matrix has a row or column which is entirely zero then its determinant is 0.
- If A is a square matrix then $det(A) = det(A^T)$.
- If B is a square matrix obtained by multiplying a row or column of A by k then $det(B) = kdet(A)$.
- If B is obtained from A by exchanging two rows then $det(B) = -det(A)$.

Slightly more work

The effect of adding a multiple of one row to another

If a matrix B is obtained from A by adding a multiple of one row to another then $det(B) = det(A)$.

An efficient algorithm for finding the determinant

Start with a square matrix A and row reduce it to a triangular matrix B using only row changes and adding multiples of one row to another. Then $det(A)$ will be $det(B)$ multiplied by $(-1)^N$ where N is the number of row changes you did and $det(B)$ can be determined by multiplying the diagonal elements of B.

Determinants of elementary matrices

Corollary (Theorem 2.2.4)

- *If* E *is an elementary matrix obtained from* I *by multiplying a row by* k *then* $\det(E) = k$.
- *If* E *is an elementary matrix obtained from* I *by interchanging two rows then* $\det(E) = -1$.
- *If* E *is an elementary matrix obtained from* I *by adding a multiple of one row to another then* $\det(E) = 1$.

Some properties

- If E is an elementary matrix then $det(EA) = det(E)det(A)$.
- (Theorem 2.3.3) A is invertible iff $det(A) \neq 0$.
- (Theorem 2.3.4) $det(AB) = det(A)det(B)$.
- (Theorem 2.3.5) If A is invertible then $det(A^{-1}) = \frac{1}{det(A)}$.

The adjoint

Definition

Suppose that A is a square matrix then the matrix of cofactors of A is

$$\begin{pmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{pmatrix}$$

Its transpose is called the adjoint of A and written $adj(A)$.

Theorem (2.3.6)

If A is invertible then

$$A^{-1} = \frac{1}{det(A)} adj(A)$$

Cramer's rule

Theorem (2.3.7)

Suppose A is invertible. Then the solution to $Ax = b$ is

$$x_1 = \frac{det(A_1)}{det(A)}, x_2 = \frac{det(A_2)}{det(A)}, \ldots, x_n = \frac{det(A_n)}{det(A)}$$

where A_j is the matrix obtained by replacing the j^{th} column of A by b.