Math 1A03 Fall 2010 Midterm 2 Formulas to remember

Derivatives

$$(x^n)' = nx^{n-1},\ n \neq 0 \qquad (a)' = 0$$

$$(e^x)' = e^x \qquad (a^x)' = a^x \ln a$$

$$(\ln x)' = \frac{1}{x} \qquad (\log_a x)' = \frac{1}{x \ln a}$$

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x \qquad (\cot x)' = -\csc^2 x$$

$$(\sec x)' = \sec x \tan x \qquad (\csc x)' = -\csc x \cot x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} \qquad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2} \qquad (\text{arccot}\, x)' = -\frac{1}{1+x^2}$$

Integrals (constants of integration are omitted)

$$\int x^n\, dx = \frac{x^{n+1}}{n+1},\ n \neq -1 \qquad \int \frac{1}{x}\, dx = \ln|x|$$

$$\int e^x\, dx = e^x \qquad \int a^x\, dx = \frac{a^x}{\ln a}$$

$$\int \sin x\, dx = -\cos x \qquad \int \cos x\, dx = \sin x$$

$$\int \tan x\, dx = -\ln|\cos x| \qquad \int \cot x\, dx = \ln|\sin x|$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| \qquad \int \csc x\, dx = -\ln|\csc x + \cot x|$$

$$\int \sec^2 x\, dx = \tan x \qquad \int \csc^2 x\, dx = -\cot x$$

$$\int \sec x \tan x\, dx = \sec x \qquad \int \csc x \cot x\, dx = -\csc x$$

$$\int \frac{1}{x^2+a^2}\, dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) \qquad \int \frac{1}{\sqrt{a^2-x^2}}\, dx = \arcsin\left(\frac{x}{a}\right)$$

Trigonometry

$\sec x = \dfrac{1}{\cos x}$

$\csc x = \dfrac{1}{\sin x}$

$\tan x = \dfrac{\sin x}{\cos x}$

$\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

$\sin(2x) = 2 \sin x \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$\sin^2 x = \dfrac{1 - \cos(2x)}{2}$

$\cos^2 x = \dfrac{1 + \cos(2x)}{2}$

$\sin x \sin y = \dfrac{\cos(x-y) - \cos(x+y)}{2}$

$\cos x \cos y = \dfrac{\cos(x-y) + \cos(x+y)}{2}$

$\sin x \cos y = \dfrac{\sin(x-y) + \sin(x+y)}{2}$

Approximate integration

$$\begin{aligned}
\int_a^b f(x)\,dx \approx M_n &= \Delta x \left[f\left(\frac{x_0 + x_1}{2}\right) + f\left(\frac{x_1 + x_2}{2}\right) + \cdots + f\left(\frac{x_{n-1} + x_n}{2}\right) \right] \\
\int_a^b f(x)\,dx \approx T_n &= \frac{\Delta x}{2} \left[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n) \right] \\
\int_a^b f(x)\,dx \approx S_n &= \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + \\
&\quad +4f(x_{n-1}) + f(x_n)] \qquad n \text{ is even}
\end{aligned}$$

Error bounds:

If $|f''(x)| \le K$ on $[a,b]$, then

$$|E_T| \le \frac{K(b-a)^3}{12n^2}, \qquad |E_M| \le \frac{K(b-a)^3}{24n^2}.$$

If $|f^{(4)}(x)| \le K$ on $[a,b]$, then

$$|E_S| \le \frac{K(b-a)^5}{180n^4}.$$